

# Liquidity Risk, Credit Risk, and Interbank Competition\*

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## Abstract

This paper examines the impact of interbank competition on liquidity risk and on the interaction between liquidity and credit risks. We first show that financial intermediation with deposit insurance may increase the impact of liquidity risk, so that intermediated loans may carry higher liquidity premia for borrowers than direct- or market-financed loans. Second, with negligible interbank competition, higher credit risk may improve loan liquidity, so a bank's need for liquidity may also induce it to take on additional credit risk. Third, we show that Bertrand competition among banks in the loan market, introduced by outside lenders purchasing the relationship-specific liquidation skill of the incumbent lender, has two potential effects: (i) it can improve loan liquidity, and (ii) it can make credit and liquidity risks comonotonic, thereby reducing the inclination of banks to take on excessive credit risk to cope with their liquidity needs. However, interbank competition improves loan liquidity only under some conditions. We identify conditions under which greater interbank competition increases loan liquidity and reduces each bank's overall risk, which includes credit and liquidity risks. On the other hand, an important implication of these results is the possible coexistence of high credit risk and high liquidity risk in the loan market if the level of credit risk taken on exceeds a certain threshold.

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# 1 Introduction

How does interbank competition affect the stability of the banking system? Banking regulators the world over have been riveted by this question, and it has been researched for decades [e.g., Allen and Gale (2004)]. Most of the literature on the relationship between market structure and banking stability has focused on the impact of competition on banks' incentives to take risk [e.g., Carletti (2007)]. Many believe that increased competition leads to socially-excessive risk taking by banks and thus jeopardizes stability [e.g., Allen and Gale (2000, 2004), and Boot and Greenbaum (1993)]. Some studies, however, suggest that the asserted relationship between competition and banks' risk taking is not robust to variations in assumptions [e.g., Caminal and Matutes (2002)], and may not even be true [e.g., Boyd and De Nicoló (2005)]. See the recent overview by Vives (2008). Risk-taking in all these papers refers to a bank's exposure to credit or default risk.

What is conspicuously missing in these analyses is liquidity risk. While it is clearly important to understand how competition affects credit risk, its effect on liquidity risk is also relevant, as the subprime lending crisis has recently shown. Many interesting questions remain unresolved on this issue: How does interbank competition affect liquidity risk? How do liquidity risk and credit risk interact? How does interbank competition affect credit risk *through* its interaction with liquidity risk, i.e., what is the effect of competition on banks' *overall* risk, which includes credit and liquidity risks?

To answer these questions, we analyze the interaction of liquidity risk with credit risk and competition. We start the basic model with a risk-free interest rate as the benchmark, then model liquidity risk under financial intermediation (i.e., lending through deposits) with complete deposit insurance. Next, we examine the impact of interbank competition on loan liquidity under various scenarios. Finally, we introduce credit risk through stochastic loan returns and analyze: (1) how banks' liquidity needs affect their credit-risk taking, and (2) how Bertrand competition among banks in the loan market affects liquidity and credit risks *jointly*. As in the previous literature, liquidity is defined as the ability to sell an asset at close to its true value. Illiquidity emerges because asset value is derived from the unique ability of the asset owner to produce cash flows, so that when the asset switches hands to someone less skilled, its value diminishes. Thus, the liquidity of an asset (a loan in our model) is measured by its market price.

We obtain four main results on liquidity risk and the impact of interbank competition on liquidity risk (direct effect) as well as credit risk (indirect effect).

First, financial intermediation, which involves financing using the bank's deposits, may actually *worsen* liquidity risk with complete deposit insurance. The reason is that when the bank acts as the intermediary between the borrowing entrepreneur and the depositors, there are *two* layers of liquidity risk for the borrower, one associated with the bank's possible liquidity shock and the other associated with the depositors' intertemporal need for liquidity

before the loan matures.

Second, interbank competition *improves* loan liquidity and lowers liquidity risk for both the borrower and the lender under some circumstances. This result depends on the existence of a market in which the liquidation technology is traded in equilibrium and can be acquired by outside (unskilled) lenders who can then compete to purchase the loan. When the loan repayment an unskilled lender can extract is below the refinancing cost, it is impossible for an unskilled lender to compete for the loan unless it can purchase the liquidation technology that can improve its liquidation skill to the level of the incumbent lender. The availability of the liquidation technology essentially leads to a competitive secondary market for the loan. That is, liquidity risk arises in the first place because a lender that experiences a liquidity need is unable to sell its loan to another lender at close to the true value of the loan, so having more viable competing lenders actually expands the potential set of buyers for the initial lender's loan. The availability of the liquidation technology increases the liquidation value that outside lenders associate with the loan, which increases the expected price at which the loan can be sold to these outside lenders, and hence improves its liquidity.

But this is not to say that competition creates liquidity unconditionally. Rather, interbank competition may reduce the market value of a liquid loan if the liquidation value of the loan to outside lenders is above the refinancing cost even without the liquidation technology. In this case, outside lenders can profit from the loan without purchasing the liquidation technology, so the market for the liquidation technology fails to exist. To put it differently, when outside lenders anticipate profits even without being equipped with the liquidation technology, they lose the incentive to purchase this technology to extract as much value as possible from the loan, and price competition consequently lowers the market value of the loan until there is zero profit from making the loan.

Third, on the issue of how liquidity risk and credit risk interact, we show that in the absence of competition, loans with higher credit risk may have *lower* liquidity risk. This may be interpreted as a put option effect created by deposit insurance. Consider two loans with equal expected payoffs: the one with higher credit risk must have higher probabilities of payoffs in both higher and lower states. Thus, given the lower-payoff truncation generated by deposit insurance, the loan with higher credit risk may have a higher *conditional* expected payoff than the loan with lower credit risk, thereby selling at a higher price and exhibiting a lower price discount due to liquidity risk. This may induce the bank to take on additional credit risk in order to cope with higher liquidity risk, which exposes an interesting tension between these two types of risks.

Finally, we show that the bank's inclination to take on additional credit risk to reduce its liquidity risk is surprisingly attenuated by interbank competition because such competition makes credit risk comonotonic with liquidity risk. The reason is that to the extent that loans with higher credit risks have higher expected liquidation values, they invite outside lenders to compete with the incumbent lender – either with or without acquiring the liquidation

technology – when the realized state is sufficiently high. This results in the loan repayment obligations in such high states being competed down to lenders’ refinancing cost, reducing the expected loan repayment and thus the incumbent’s surplus from increasing credit risk to achieve a higher expected loan resale value. A possible unfavorable outcome is the coexistence of high credit risk and high liquidity risk. For example, the subprime crisis.

To summarize, our main contribution to the literature is to highlight the interaction between liquidity and credit risks and the effect of interbank competition on this interaction. Liquidity risk creates adverse consequences for both the borrower and the lender: the borrower bears costs of illiquidity, including a higher return premium demanded by the lender, possible early liquidation, and the risk of receiving no financing, while the lender faces an inability to sell the loan at its full value and may even take on higher credit risk in order to cope with its liquidity need. Interbank competition gives outside lenders incentives to acquire the skill to extract value from the loan, and can thus possibly improve loan liquidity, which in turn reduces liquidity risk for both the borrower and the lender. Moreover, through the interaction between liquidity risk and credit risk, competition can restore the appropriate credit-risk incentive for the bank. As a result, the overall risk exposure of the bank may be diminished by the introduction of competition as long as the level of credit risk is *not* too high.

This paper is related to the large literature on interbank competition and its impact on stability and regulation in banking [e.g., Allen and Gale (2000, 2004), Besanko and Thakor (1993), Boot and Greenbaum (1993), Boot and Marinč (2007), Boyd and De Nicoló (2005), Boyd, De Nicoló and Jalal (2007), Caminal and Matutes (1997, 2002), Cordella and Yeyati (2002), Edwards and Mishkin (1995), Keeley (1990), Matutes and Vives (1996, 2000), and Petersen and Rajan (1995)]. Vives (2008) provides a nice introduction to this literature, while Carletti (2008) and Degryse and Ongena (2008) survey the relevant theoretical and empirical strands of this literature, respectively. This literature has focused mainly on the impact of competition on the credit risk taken on by banks. Where we differ is in our focus on liquidity risk and the impact of interbank competition on the interaction between liquidity and credit risks.

This paper is also related to the financial intermediation literature on liquidity [e.g., Acharya, Gromb and Yorulmazer (2007a, 2007b), Acharya and Schaefer (2006), Bhattacharya and Thakor (1993), Calomiris and Kahn (1991), Chari and Jagannathan (1988), Diamond and Dybvig (1983), Diamond and Rajan (2001), Gale and Vives (2002), Goldstein and Pauzner (2005), Gorton (1985, 1988), Gorton and Winton (2000), Jacklin and Bhattacharya (1988), Kashyap, Rajan and Stein (2002), Postlewaite and Vives (1987), Rochet and Vives (2004), and Wagner (2008)].<sup>1</sup> Our main point of departure relative to this

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<sup>1</sup>Our paper differs from the literature that focuses on liquidity shocks faced by borrowing firms rather than financial institutions [e.g., Acharya and Viswanathan (2007) and Holmström and Tirole (1998)], as well as the asset pricing implications of liquidity risk [e.g., Acharya and Pedersen (2005)]. In Acharya and

literature is our additional consideration of credit risk and interbank competition.

Our model is built on the same argument as in Diamond and Rajan (2001) that loans are illiquid because of the uniqueness and non-transferability of the incumbent relationship lender's skill in extracting loan repayments from the borrower. There are, however, some important modeling differences between our paper and Diamond and Rajan (2001). First, unlike their model, we analyze the role of competition among banks by introducing a market in which the technology with which the lender extracts repayments from the borrower (i.e., the "liquidation technology") can be traded. This market serves as the mechanism by which interbank competition affects liquidity risk. Second, whereas Diamond and Rajan (2001) consider a world with no deposit insurance, our liquidity model is with complete deposit insurance. This allows us to leave the sequential service constraint (SSC) out of the picture, facilitating a relatively clean analysis of the impact of competition on liquidity. Moreover, it has already been shown that financial fragility – arising in part from the lack of deposit insurance – encourages liquidity creation, so to the extent that competition increases fragility, there would be potentially two channels through which competition would affect liquidity, one being a direct effect and the other indirectly through fragility. This would make it difficult to isolate the extent to which interbank competition impacts liquidity, independently of fragility. Third, we introduce credit risk as a distinct source of risk for the bank, which permits us to analyze the simultaneous effect of competition on both credit and liquidity risks.

In terms of results, the sharpest contrast between the two papers can be described as follows. First, while Diamond and Rajan (2001) focus on liquidity risk, we show how liquidity risk can increase credit risk in the absence of competition. Second, while Diamond and Rajan's (2001) main message is that bank fragility is what is needed to induce banks to create liquidity in the absence of deposit insurance, our main message is that when deposit insurance is *de facto* complete, it is interbank competition that causes liquidity to be created.

The rest of the paper is organized as follows. In Section 2, we first lay out the basic model with neither competition nor deposit financing, derive some basic results about liquidity risk, and then extend the model to a setting of financial intermediation with complete deposit insurance. We add interbank competition to the model and analyze its impact on liquidity creation in Section 3. Whereas there is no credit risk in Sections 2 and 3, we introduce credit risk through payoff uncertainty in Section 4 and reexamine the nature of liquidity risk and the role of interbank competition. Empirical predictions are discussed in Section 5. We conclude in Section 6 with policy implications and a few remarks on what our analysis says about the subprime crisis. All proofs are in the Appendix.

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Viswanathan (2007), for example, liquidity risk arises because the firm may suffer a liquidity shock but moral hazard prevents it from borrowing more to meet the shock. This forces liquidation of assets.

## 2 Analysis of Liquidity Risk

Liquidity risk is the risk that an asset owner will not be able to realize the full value of that asset at the time a sale is desired. Both lenders and borrowers are exposed to liquidity risk. To lenders, it means the potential inability to sell a loan or raise money against it to the full value of the expected repayments. From a borrower's perspective, it is the fear of not receiving financing or getting a loan with terms that elevate the cost of financing.

When a lender recognizes that a random liquidity shock may hit with a positive probability and that in that event it will be unable to sell the loan at its full value, it will price the loan so that the expected cost associated with this liquidity shock is passed on to the borrower. As a result, the borrower bears costs of illiquidity, which include the higher cost of borrowing due to the impact of liquidity risk on the price of the loan, possible early liquidation of his project, and in the worst case an inability to obtain financing.

In this section, we start with a model in which there is a riskless project financed through direct lending. We show that even loans that have zero credit risk have liquidity risk. Then, we examine how liquidity risk is affected by the introduction of financial intermediation with complete deposit insurance. We are able to show that liquidity risk may actually be exacerbated by intermediation in a setting without interbank competition.

### 2.1 The Basic Model

The basic model adapts Diamond and Rajan's (2001) model by adding a risk-free interest rate, which is simply an opportunity cost for now but proves important in later analysis. We will be brief when dealing with results already established in their paper.

#### 2.1.1 The Framework

**Economy:** Consider an economy in which all agents are risk-neutral. There exists a risk-free asset of unlimited supply that returns  $R_f = 1 + r_f$  each period for every dollar invested, where  $r_f (> 0)$  is the risk-free interest rate. Agents are of two types: entrepreneurs and potential financiers. Assume that there are many potential financiers, so the economy has no aggregate shortage of liquidity.

**Entrepreneurs:** Each entrepreneur has a project that lasts for two periods and three dates (i.e., period 1 is from date 0 to date 1 and period 2 is from date 1 to date 2). The project requires an investment of \$1 at date 0. If the entrepreneur works on his project, it produces a riskless cash flow of  $C_1$  at date 1 and  $C_2$  at date 2. The entrepreneur does not have money to finance his project, so he raises funds by taking a loan of \$1 from a lender. The loan contract specifies a repayment  $Z_t$  that the borrower is required to make at date  $t$ . We can consider  $Z_1$  as the interim-period interest payment and  $Z_2$  as the principal repayment plus interest at the end of the loan contract. For simplicity, we assume that

there is no entrepreneurial project choice, so there is no project-choice moral hazard of the sort examined in Boyd and De Nicoló (2005).

**Financiers:** We call the lender who invests in the project at date 0 the relationship lender. This lender develops specific skill in identifying the liquidation value of the entrepreneur's assets through its relationship with the entrepreneur. As a result, its advantage over other lenders is that it can identify the second-best use of the assets better than anyone else. More formally, at time  $t$ , the relationship lender can put the assets in an alternative use to generate  $X_t$  (i.e., the "liquidation value" of the assets), while other lenders, called "unskilled lenders," can generate only  $\beta X_t$ , where  $0 \leq \beta < 1$ . Since it takes time and effort for a lender to become knowledgeable about the entrepreneur's business, one entrepreneur can have only one relationship lender. Note that  $C_t$ ,  $X_t$ , and  $\beta$  are all publicly known at date 0, so there is no information asymmetry about cash flows and liquidation values.

**Limited Commitment:** One important feature of this model is limited commitment, and it has both a temporal aspect and a cross-sectional aspect. The *temporal aspect* is that at any date the entrepreneur can commit to using his specific skill to work on the project *only* for that date. This implies that after borrowing and investing at date 0, the entrepreneur can threaten to quit before the cash flow is produced at date 1 unless the terms of financing are renegotiated. He can do this again before date 2. For simplicity, we assume that the entrepreneur has all the bargaining power. If the entrepreneur renegotiates his loan contract and the relationship lender accepts the revised schedule of payments, the entrepreneur produces that date's cash flow, makes the spot payment required by the revised schedule, and continues in possession of the assets. But if the relationship lender rejects the revised contract, the cash flow is not produced that period and the lender takes possession of the assets with the choice to liquidate the assets.

Meanwhile, the relationship lender has the option to sell the date-2 portion of the loan to another lender at date 1. By the assumption that only the initial lender is skilled, we know that the buyer of the loan must be unskilled, which highlights the role of the *cross-sectional aspect* of limited commitment: the relationship lender *cannot* commit to using its specific liquidation (extraction) skill on behalf of *other* lenders (including the buyer of the loan) at any future date.

**Liquidity Shock:** With probability  $\theta$  at date 1, the relationship lender gets a liquidity shock – an investment or consumption opportunity that is valued higher than the risk-free asset. This shock makes one unit of date-1 goods worth  $R_l (> R_f)$  units of date 2 goods to the lender, i.e., the discount factor changes from  $\frac{1}{R_f}$  to  $\frac{1}{R_l}$  when the shock hits. We refer to a lender that receives a liquidity shock as impatient or  $I$ , and a lender that does not receive a liquidity shock as patient or  $P$ . The realization of the liquidity shock

is the relationship lender's private information. However, using the Revelation Principle, a menu of loan contracts can be designed such that the lender truthfully reports this private information via its choice of loan contract. We can thus view the loan contract as setting repayments  $Z_t$  contingent on the realized (and truthfully reported) liquidity shock to the lender. Meanwhile, there exist numerous unskilled lenders that do not suffer a liquidity shock and are able to buy the loan from the relationship lender.

Further, we make the following assumptions on the project's payoffs (cash flows) and asset liquidation values:

- Assumption 1:  $\min \left[ C_1 + \frac{C_2}{R_f}, \frac{C_1 + C_2/R_f}{X_1} \right] > R_l > R_f > 1$ .
- Assumption 2:  $C_2 > X_2$ .
- Assumption 3:  $\max \left[ \frac{X_1}{R_f}, \frac{X_2}{R_f^2} \right] \geq 1$ .

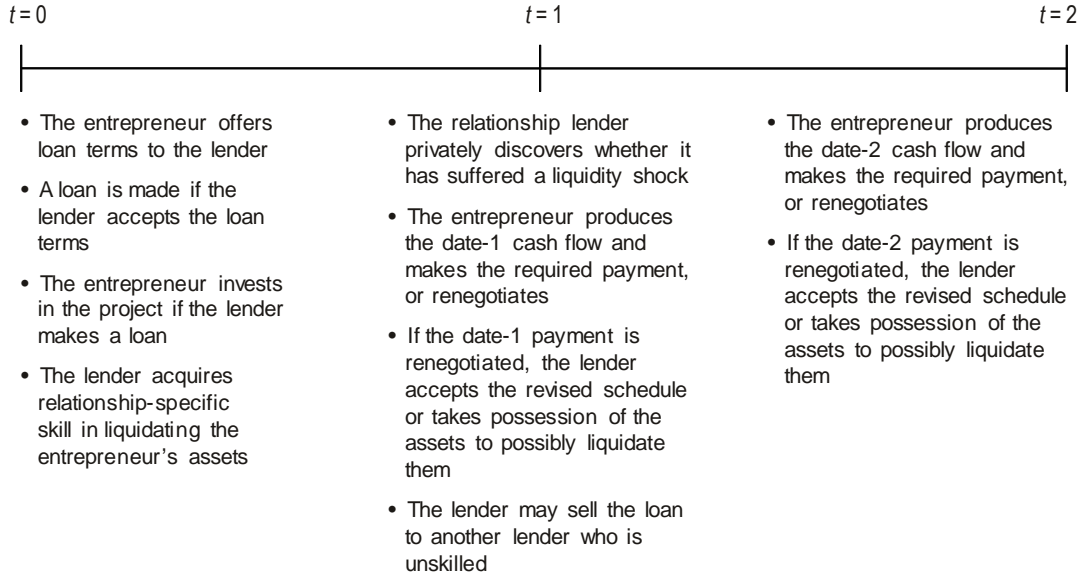
These assumptions basically make the project worth pursuing for the entrepreneur at both dates and worth financing for the relationship lender if it is patient. Assumption 1 says that the cash flows produced by the project, valued at the opportunity cost of either \$1 at date 0 or  $X_1$  at date 1, provide higher returns than the relationship lender's discount rate even when the lender suffers a liquidity shock. Thus, investing in the project is efficient for the entrepreneur. Assumption 2 shows that the date-2 cash flow is higher than the liquidation value, and hence, the project is worth continuing at date 2. Assumption 3 indicates that the date-0 present value of the maximum amount the patient relationship lender can extract from the loan – the amount that the entrepreneur can commit to paying the lender – is no less than the investment, so the project can be financed with a patient relationship lender.

*Figure 1* summarizes the sequence of events in this basic model.

### 2.1.2 Analysis

In this subsection, we discuss: (1) the liquidity premium that the entrepreneur must pay to receive financing, (2) the implications of the loan contract being renegotiation-proof, (3) the order of the entrepreneur's preferences regarding the relationship lender's actions, and (4) the conditions for the relationship lender to truthfully reveal its type (patient or impatient). These results are sustained when we add more structure to the model. Finally, we solve for the condition under which the entrepreneur obtains financing for his project *and* the relationship lender sells the loan when hit with a liquidity shock.

**Liquidity Premium:** Given risk neutrality, the entrepreneur's objective is to maximize his expected profit. Borrowing funds and investing in the project is a weakly dominant strategy for the entrepreneur. In order for his project to be financed with a particular loan



*Figure 1. Sequence of Events (The Basic Model)*

contract, the entrepreneur must satisfy the relationship lender's individual rationality (*IR*) and incentive compatibility (*IC*) constraints.

Let the relationship lender's reservation utility be  $\underline{U}$ . If valued at date 2,  $\underline{U} = \theta R_f R_t + (1 - \theta) R_f^2$ , given the probability  $\theta$  of a liquidity shock. Hence, the entrepreneur must give the relationship lender at least  $\underline{U}$  at date 2 to satisfy the lender's *IR* constraint. With the entrepreneur maximizing his expected payoff, we also see that the *IR* constraint is binding in equilibrium.

Define the liquidity premium ( $\mathcal{LP}$ ) as the premium in expected payments (valued at date 2) that the entrepreneur has to make over the expected payments the lender would have received by investing in the risk-free asset at date 0. Let  $V$  be the expected payments (also valued at date 2) made by the entrepreneur in all states other than the state in which the lender receives a liquidity shock, and  $V_1^*$  be the date-1 cash flow to the impatient lender that has received a liquidity shock, including the payment by the entrepreneur and proceeds from the loan sale or project liquidation. Then the liquidity premium is  $V + \theta V_1^* R_f - R_f^2$ . The lender's *IR* constraint says that  $V + \theta V_1^* R_t = \underline{U}$ . Solving  $V$  from the *IR* constraint and substituting it into the liquidity premium, we get the liquidity premium as:

$$\mathcal{LP} = \theta (R_t - R_f) (R_f - V_1^*). \quad (1)$$

We can see from (1) that if the impatient lender can receive a cash flow  $V_1^*$  at date 1 that equals  $R_f$ , then  $\mathcal{LP} = 0$  and the entrepreneur obtains financing without paying a liquidity premium. We define a "liquid loan" as one that involves  $\mathcal{LP} = 0$ . If the total cash available at date 1 for the entrepreneur to pay an impatient lender is below  $R_f$ , the loan has a positive liquidity premium and is called illiquid, and the entrepreneur can possibly

face liquidation and even denial of credit. As shown in Diamond and Rajan (2001) and also here a little later, with the lender's liquidity need being private information, the liquidity premium is always nonnegative ( $\mathcal{LP} \geq 0$ ), i.e., liquidity insurance does not exist.

**Renegotiation-proof Contract:** Let us examine the upper limits of how much the entrepreneur can commit to paying at both dates.

At date 2, the entrepreneur will pay no more than  $X_2$  to the relationship lender, knowing that the maximum it can get at this time is  $X_2$  if the date-2 cash flow is not produced and the assets are thereafter liquidated. Note that the relationship lender cannot expect to liquidate the assets at any price above  $X_2$ .

At date 1, if the relationship lender is patient and the entrepreneur initiates renegotiation, the lender can accept the revised contract or choose to liquidate at either date 1 to get  $X_1$  or date 2 to get  $X_2$ , depending on whether  $X_1$  or  $\frac{X_2}{R_f}$  (the present value of  $X_2$  at date 1) is higher. Selling when patient is a weakly dominated strategy for the relationship lender because it has the best liquidation skill. Thus, the date-1 present value of the amounts the entrepreneur will commit to paying over two dates will not exceed  $\max\left[X_1, \frac{X_2}{R_f}\right]$ .

Suppose now that the relationship lender receives a liquidity shock at date 1 and is therefore impatient. If the entrepreneur attempts to renegotiate, the lender can get  $X_1$  by liquidating the assets at date 1 or a date-1 present value of  $\frac{X_2}{R_l}$  by waiting until date 2 to liquidate. A third option is to sell the loan contract to an unskilled lender that has not suffered a liquidity shock. Let  $S$  be the maximum selling price of the loan that the buyer is willing to pay, which is equivalent to the date-1 present value of the maximum amount that the buyer can collect on the loan at date 2. It is straightforward to see through backward induction that  $S = \frac{\beta X_2}{R_f}$ .<sup>2</sup>

In summary, the largest date-1 present value that the impatient relationship lender can extract is  $E_1^I \equiv \max\left[X_1, S, \frac{X_2}{R_l}\right]$ , and the maximum amount that the patient lender can extract is  $E_1^P \equiv \max\left[X_1, \frac{X_2}{R_f}\right]$ . Clearly,  $E_1^P \geq E_1^I$  and  $E_1^P R_f \leq E_1^I R_l$ .

Now, let us examine the maximum amount of cash that is actually available to pay the relationship lender at date 1. If the loan is not liquidated, the relationship lender can get up to  $C_1 + S$  if it collects all of the date-1 cash flow and sells the loan. If the loan is liquidated, it gets  $X_1$ . Note that a loan contract can specify the conditions under which the loan can be liquidated. Given that liquidation is not allowed, the impatient lender can extract no more than  $\min\left[C_1 + S, E_1^I\right]$  at date 1.

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<sup>2</sup>The buyer of the loan, being unskilled at liquidating the entrepreneur's assets, can obtain a maximum renegotiation-proof payment of  $\beta X_2$  at date 2. Even if the loan buyer hires the relationship lender to collect the payment or liquidate the assets, it will still receive no more than  $\beta X_2$  at date 2 because the relationship lender cannot commit to use its specific liquidation skill on the behalf of others. Thus, the loan is worth no more than  $\frac{\beta X_2}{R_f}$  to an unskilled lender at date 1.

**Entrepreneur's Order of Preferences:** Let  $V_t^j$  be the cash paid by the entrepreneur to the relationship lender of type  $j$  at date  $t$ , where  $j \in \{I, P\}$  and  $t \in \{1, 2\}$ ; recall that  $I$  denotes an impatient lender and  $P$  denotes a patient lender. The entrepreneur chooses these cash repayments to maximize his expected profit  $\Phi$ . That is, he solves

$$\max_{V_1^I, V_2^I, V_1^P, V_2^P} \Phi. \quad (2)$$

To understand the entrepreneur's expected profit  $\Phi$ , we note that  $\Phi$  depends on the lender's type (patient or impatient) and whether the entrepreneur's date-1 cash flow is enough to repay the lender and thereby avoid liquidation. When there is no liquidation, the entrepreneur's expected profit with a type- $j$  lender can be written as:  $C_1 R_f$  (the date-1 cash flow compounded for one period at  $R_f$ ) plus  $C_2$  (the date-2 cash flow) minus  $V_1^j R_f$  (the date-1 repayment obligation to the type- $j$  lender compounded at  $R_f$ ) minus  $V_2^j$  (the date-2 repayment obligation to the type- $j$  lender), i.e.,  $C_1 R_f + C_2 - V_1^j R_f - V_2^j \equiv \pi_n^j$ . When there is liquidation, the project liquidation value at date-1 is  $X_1$ , so the entrepreneur keeps  $X_1 - V_1^j$  after repaying the type- $j$  lender, and the entrepreneur's expected profit is  $(X_1 - V_1^j) R_f \equiv \pi_l^j$  when it is compounded for one period.<sup>3</sup>

Since  $\theta$  is the probability of an impatient lender and  $1 - \theta$  is the probability of a patient lender, we can construct the following table:

Table 1. *Repayment Obligations, Cash Flows, and Entrepreneur's Expected Profit*

Repayment Obligations and Cash Flows	$\Phi$
$V_1^I, V_1^P \leq C_1$ (no liquidation in all states)	$\theta \pi_n^I + (1 - \theta) \pi_n^P$ (2a)
$V_1^I > C_1, V_1^P \leq C_1$ (liquidation only when lender is of type $I$ )	$\theta \pi_l^I + (1 - \theta) \pi_n^P$ (2b)
$V_1^I \leq C_1, V_1^P > C_1$ (liquidation only when lender is of type $P$ )	$\theta \pi_n^I + (1 - \theta) \pi_l^P$ (2c)
$V_1^I, V_1^P > C_1$ (liquidation in all states)	$\theta \pi_l^I + (1 - \theta) \pi_l^P$ (2d)

Based on  $\Phi$ , we see that the entrepreneur always prefers a lower liquidity premium to a higher premium, no liquidation to liquidation, and financing to no financing. We write the order of his preferences as follows:

1. First, the entrepreneur would like to have a liquid loan with no liquidity premium.

<sup>3</sup>Two things need to be noted here. First, if an impatient lender sells the loan, the proceeds from the loan sale are considered as  $V_2^I$  because the portion of the loan sold consists of only the date-2 payment. Second, we do not need to consider the possibility of liquidation at date 2 because of Assumption 2 ( $C_2 > X_2$ ).

2. If the loan is illiquid, the entrepreneur would prefer to pay as small a liquidity premium as possible. To accomplish this, he always prefers the impatient lender to sell the loan at date 1 to maximize liquidity.
3. If selling the loan does not satisfy the lender's *IR* constraint, the entrepreneur prefers the impatient lender to keep the loan at date 1, which results in a higher liquidity premium with  $V_1^* \leq C_1 < C_1 + S$ .
4. If the lender's *IR* constraint is still not satisfied, the entrepreneur allows the impatient lender to liquidate, which results in a lower expected profit from the project.
5. If none of the above satisfies the lender's *IR* constraint, the entrepreneur receives no financing and foregoes the project.

**Revelation Mechanism:** The entrepreneur's profit maximization is subject to conditions that make payments: (1) possible to extract (i.e., within the liquidity bound), (2) renegotiation-proof, (3) individually rational, and (4) incentive compatible. We have already discussed the first three types of constraints, so now we examine the incentive compatibility (*IC*) constraints.

The loan contract serves as a revelation mechanism. With the lender's liquidity need being private information, the entrepreneur offers a menu of contracts with each element being contingent on the realized type of the lender. That is, in the spirit of the Revelation Principle [e.g., Myerson (1979)], the lender reports its type to the entrepreneur at date 1 and receives the loan contract "designated" for that type. The menu of loan contracts from which the lender receives a contract contingent on its report must be designed so that the lender will report its type truthfully.

This rules out the possibility of liquidity insurance. Suppose that the entrepreneur pays the impatient lender more than  $R_f$  at date 1, which results in a negative liquidity premium ( $\mathcal{LP} < 0$ ), i.e., liquidity insurance. It makes the present value of payments that the patient lender can receive drop below  $R_f$  at date 1 due to the binding *IR* constraint. Then, the patient lender will always report itself as impatient and the mechanism fails. Thus,  $V_1^* \leq R_f$  and  $\mathcal{LP} \geq 0$ .

The most effective way for the entrepreneur to satisfy the *IC* constraints is back-loading payments to the patient lender and front-loading payments to the impatient one. That is, at date 1 the entrepreneur pays the impatient lender as much as possible but nothing to the patient type, and compensates the patient lender with a larger payment at date 2.

**Financing Condition:** Based on the analysis above, it is easy to see that the condition for the entrepreneur to be financed at date 0 with a *liquid* loan is that the maximum cash available to pay the impatient lender at date 1 equals or exceeds  $R_f$ , that is,

$$L \equiv \min [C_1 + S, E_1^I] \geq R_f. \quad (3)$$

We thus assume  $\min [C_1 + S, E_1^I] < R_f$  to explore the more interesting case in which the loan is *illiquid*. Furthermore, we make the following assumptions to focus on selected cases based on project payoffs and asset values instead of all possible scenarios:

- Assumption 4:  $\min [C_1 + S, E_1^I] = C_1 + S < R_f$ .
- Assumption 5:  $(C_1 + S) R_l < \underline{U}$ .

The implications of Assumption 4 are two-fold. First, because the maximum liquidity that is available at date 1 for the entrepreneur to pay the impatient lender ( $C_1 + S$ ) is less than the amount the entrepreneur can commit to paying ( $E_1^I$ ), the impatient lender will receive  $C_1 + S$  at date 1. Second, the loan is illiquid because  $C_1 + S < R_f$ . Assumption 5 excludes the possibility that the relationship lender's *IR* condition will be satisfied if the loan contract offers a total date-2 value of  $(C_1 + S) R_l$  to the lender, regardless its type.

For an illiquid loan to be made, the entrepreneur specifies in the contract whether the relationship lender can sell, keep, or liquidate the loan if hit with a liquidity shock. In the subsequent analysis, we provide only the condition for the loan to be sold by the impatient lender at date 1. This is the case in which the entrepreneur pays the smallest possible liquidity premium. Also, it is the most relevant to the market value of the loan ( $S$ ), which we define as the loan liquidity later. Conditions for the loan to be kept and liquidated by the impatient lender can be derived in a similar way and hence are not repeated here.<sup>4</sup>

**Lemma 1** *With Assumptions 1-5, if the relationship lender's type (whether it receives a liquidity shock or not) is the lender's private information and the loan contract is made contingent on the lender's reported type, then the entrepreneur will be financed at date 0 with a positive liquidity premium and the impatient relationship lender will sell the loan at date 1 if the following condition holds:*

$$E_1^P \geq \underline{E}_1 \equiv R_f + \frac{\theta}{1 - \theta} \frac{R_l}{R_f} [R_f - (C_1 + S)] \text{ and } C_1 + S \geq \underline{CS}_1 \equiv \frac{E_1^P R_f}{R_l}. \quad (\text{Condition 1})$$

Lemma 1 shows that there exist lower bounds on the cash payoffs and asset values of the project for it to be financed with a positive liquidity premium even though it has no default risk and ought to be financed in the absence of liquidity concerns. Note that if Condition 1 does not hold, the loan cannot be sold to satisfy the relationship lender's *IR* and/or *IC* constraint, so the entrepreneur indeed faces a higher liquidity premium and possibly early liquidation or denial of credit. Thus, it verifies the existence of liquidity risk when the lender faces a likelihood of needing liquidity before the loan matures, even if the likelihood is slim and the assets have no default risk.

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<sup>4</sup>The full solution of this financing game is available upon request.

## 2.2 Financial Intermediation

We now add financial intermediation to the basic model by assuming that loans are financed with deposits. What we want to know is how liquidity risk is affected by intermediation. We consider both the non-banking and banking outcomes, and show that financial intermediation increases liquidity risk when deposits are fully insured.<sup>5</sup>

### 2.2.1 Deposit Contract and Insurance

Let the relationship lender be a bank that "borrows against the loan." That is, the relationship lender raises \$1 from depositors at date 0 to invest in the project. The deposit contract promises depositors an interest payment of  $r_f$  at date 1 and interest plus principal repayment of  $R_f = 1 + r_f$  at date 2. Thus, we see that deposit maturity matches the loan maturity perfectly. Another important feature of the deposit contract is that funds can be withdrawn at depositors' demand in the interim period, i.e., date 1. We assume that a proportion  $\eta$  of deposits is expected to be withdrawn at date 1.

Deposits are fully insured. If the bank fails to make the promised payment at either date *or* if it fails to fully repay at withdrawals, the deposit insurer pays off the depositors and closes the bank. For simplicity, we further assume that the bank has no equity capital and cannot raise external funds other than selling the loan to make the interest payment and satisfy the withdrawals at date 1. That is, payments from the project are the only source of liquidity for the bank at date 1. However, once the bank survives the first period, it will be able to raise additional capital through either deposits or equity at date 2 if necessary. Thus, the bank is only concerned about its default risk at date 1, and the total liquidity needed for solvency is  $r_f + \eta$  considering both the promised interest and the expected withdrawals.

With Assumptions 1-3, problems may indeed reside in the date-1 payment. If  $C_1 < r_f + \eta$ , the bank must try to sell or liquidate the loan to cover the interest payment and expected withdrawals at date 1 even if it is patient. But, if  $C_1 + S < r_f + \eta$  and  $X_1 < r_f + \eta$ , then the entrepreneur receives no financing because the bank will certainly default at date 1 if it makes the loan. In order to rule out the need for a *patient* lender to sell or liquidate the loan and also eliminate the risk that such a lender will default, we assume:

- Assumption 6:  $C_1 \geq r_f + \eta$ .

Note that two features of the model are crucial: (1) not only does the bank pay some interest to depositors at date 1, but there is also an expected level of early withdrawals; and (2) there are no sources of funds for the bank other than those obtained from selling the loan to pay depositors the promised interest or demanded withdrawals at date 1.

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<sup>5</sup>This is in contrast to Diamond and Rajan (2001) who consider only the intermediated credit market with banks and conclude that a fragile capital structure subject to runs commits banks to creating liquidity.

Both features are reasonable. First, deposit contracts that require interest payments before maturity are common. One reason for this feature may be that it provides interim verification to depositors that the bank is solvent; in this case, interest would be reinvested.<sup>6</sup> Another reason may be that depositors have intertemporal consumption needs that are satisfied with interest payments; in this case, interest would be consumed. Even if no interim-period interest payments are promised, depositors, like banks, can experience liquidity shocks and demand immediate withdrawal of their deposits. Thus, even when banks do not need to pay interest before maturity, they will need to reserve liquidity for an expected level of withdrawals. Our results hold as long as  $r_f + \eta > 0$ , and it is not necessary for both  $r_f$  and  $\eta$  to be positive. Next, the justification for the second assumption is that banks can be financially constrained, just like firms. When all the available funds are invested, banks may not be able to raise any more debt or equity.

Now, why do we lift the financial constraint on the bank during the second period? While we can justify doing so by observing that investors are assured of the bank's repayment capability after its survival in the first period, so they are willing to provide second-period financing to the bank, the main purpose of removing this restriction is to simplify. If the project continues to be the only resource of liquidity for the bank, we must assume  $C_1 + S - r_f - \eta \geq \frac{(1-\eta)R_f}{R_l}$  in order for the impatient lender to have the option to sell the loan at date 1 while staying solvent through date 2. Suppose this is not true. Then the impatient lender has to keep the loan at date 1, which leads to a higher liquidity premium. This even strengthens the result we are about to show.

The time line in this setting is similar to the one shown in *Figure 1*, with the additions of the financial intermediation features described above.

### 2.2.2 Analysis

First, the condition for having a liquid loan through intermediation remains the same as in the basic model since the liquidity premium for this model is computed using the same formula as for the basic model, i.e., Equation (1).

Next, it is simple to solve for the condition under which an illiquid loan can be made and the relationship lender will sell the loan when hit with a liquidity shock. While all the constraints in the basic model remain, we now must satisfy a lower bound on the date-1 payment. That is,  $V_1^j \geq r_f + \eta$ ,  $j \in \{I, P\}$ . It does not impose an additional constraint on the payments to the impatient lender since these payments are front-loaded, but it does constrain the payments to the patient lender. Now, payments to the patient lender can no longer be completely back-loaded.

The relationship lender's *IR* constraint and the patient lender's *IC* constraint can be satisfied as before. However, the *IC* constraint for the impatient lender become harder to satisfy. For the impatient lender to sell the loan at date 1, its *IC* constraint is  $(C_1 + S) R_l \geq$

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<sup>6</sup>The reinvestment of interest by depositors may be viewed as a new and separate deposit contract.

$V_1^P R_f + V_2^P$ . With the basic model, we set  $V_1^P = 0$  to solve for the lowest required date-1 present value of loan repayments to the relationship lender, i.e.,  $C_1 + S$  and  $E_1^P$ . With this model, we must set  $V_1^P = r_f + \eta$  to find the solution. Thus, we expect to see a higher required  $C_1 + S$  relative to  $E_1^P$  and fewer circumstances in which the loan can be sold by the impatient lender. This then implies a higher liquidity premium charged to the entrepreneur and a lower expected profit from the project since the loan will have to be kept or liquidated by the impatient lender, or worse, not be made at all.

The following lemma shows that our intuition is indeed correct.

**Lemma 2** *With Assumptions 1-6, if lending is provided through financial intermediation with complete deposit insurance as described above, then the entrepreneur will be financed at date 0 with a positive liquidity premium and the impatient relationship lender will sell the loan at date 1 if the following condition holds:*

$$E_1^P \geq \underline{E}_2 \equiv R_f + \frac{\theta}{1-\theta} \frac{R_l}{R_f} [R_f - (C_1 + S)] \quad \text{and}$$

$$C_1 + S \geq \underline{CS}_2 \equiv \frac{E_1^P R_f}{R_l} + (r_f + \eta) \left(1 - \frac{R_f}{R_l}\right). \quad (\text{Condition 2})$$

Similar to Lemma 1, Lemma 2 also provides restrictions on the project's cash payoffs and asset values in order for the project to be financed in the setting with financial intermediation. Condition 2 is more stringent than Condition 1, its counterpart in the basic model without financial intermediation. We formalize this result as follows:

**Proposition 1** *Under financial intermediation with complete deposit insurance, an illiquid loan is sold by an impatient relationship lender at date 1 in weakly fewer circumstances than without financial intermediation, which results in a weakly higher liquidity premium and a weakly lower expected profit from the project for the entrepreneur.*

Proposition 1 tells us that the entrepreneur may face higher liquidity risk under financial intermediation than if it obtained non-intermediated credit, e.g., directly from the capital market. The intuition is that when the bank acts as the intermediary between the entrepreneur (the borrower) and the depositors (the actual lender), liquidity risk is associated with not only the bank's liquidity shock but also the depositors' liquidity needs. That is, there are two layers of liquidity risk, one associated with the "so-called" lender (i.e., the bank) and the other associated with the actual lender (i.e., the depositors). In this setup, although the depositors' liquidity needs in the interim period are the sum of a deterministic interest payment and an expected level of withdrawals ( $r_f + \eta$ ), the results would not be qualitatively different if their liquidity needs were stochastic.

### 3 Interbank Competition

We have shown that the financier's liquidity need is costly to the entrepreneur and it is even more so when lending occurs through financial intermediaries with deposit insurance. How can the liquidity risk faced by the entrepreneur be reduced? The condition for having a liquid loan is the same whether the project is funded through direct lending or insured deposits, that is,  $L = \min [C_1 + S, E_1^I] \geq R_f$  [Equation (3)].

The source of loan illiquidity is that the maximum loan sale price ( $S$ ) is lower than the present value of the payment the relationship lender can extract at date 2 ( $X_2$ ), and it arises because the relationship lender cannot commit to using its liquidation skill on behalf of the loan buyer. Since an unskilled lender can extract only  $\beta X_2$  at date 2, we have  $S = \frac{\beta X_2}{R_f}$ .

Based on this argument, the loan will always be liquid if  $S = \frac{X_2}{R_f}$ .<sup>7</sup> We can easily see that  $S = \frac{X_2}{R_f}$  is a sufficient condition to satisfy  $L \geq R_f$ , but not necessary. The condition  $L \geq R_f$  can be satisfied by increasing  $S$  to some value below  $\frac{X_2}{R_f}$ . We thus define the "liquidity of a loan" as the maximum market price of the loan when sold. We show below that as interbank competition increases, loan liquidity approaches  $\frac{X_2}{R_f}$  under some conditions, but never equals it, even asymptotically. Nonetheless, an initially illiquid loan may become liquid in the presence of interbank competition.

#### 3.1 Competitive Environment

Now, we introduce competition into the model to formally examine the effect of interbank competition on liquidity risk.

##### 3.1.1 Intensity of Competition

Suppose that the incumbent bank (i.e., the relationship lender) faces  $N$  ( $\geq 2$ ) competing banks that are: (1) unskilled at extracting value from the loan,<sup>8</sup> and (2) not hit with a liquidity shock at date 1. We assume that the entrepreneur searches for potential, alternative lenders and each of the  $N$  unskilled banks independently has a probability  $\lambda$  ( $> 0$ ) of being found by the entrepreneur.<sup>9</sup> Thus, the expected number of competing banks found by the entrepreneur is  $\lambda N$ , and the probability of finding at least one competing bank is:

$$\Lambda \equiv 1 - (1 - \lambda)^N. \quad (4)$$

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<sup>7</sup>If  $S = \frac{X_2}{R_f}$ , we have  $E_1^I = E_1^P$ . Then,  $\min [C_1 + S, E_1^I] = \min \left[ C_1 + \frac{X_2}{R_f}, E_1^P \right]$ , which is the maximum date-1 present value a patient lender can extract and hence cannot be below  $R_f$  for lending to occur.

<sup>8</sup>That is, competing banks can receive only a lower value from liquidating the entrepreneur's assets compared to the relationship lender.

<sup>9</sup>This formulation, consistent with a spatial model, is similar to that in Boot and Marinč (2007). What is different in our approach is that we specify  $\lambda$  as the probability that each of the competing banks is being located by the borrower, whereas they define a probability  $q$  of successfully finding one competitive offer.

$N$  and  $\lambda$  are parameters that measure the intensity or degree of competition in that the interbank loan market becomes more competitive as  $N$  and  $\lambda$  increase.<sup>10</sup>

Once the entrepreneur finds a second lender, the incumbent bank and the second lender compete as Bertrand competitors. Hence, competition occurs with probability  $\Lambda$ , which is increasing in both  $N$  and  $\lambda$ . There will be more than two banks competing if the entrepreneur locates more than one competitor. We assume that the number of competitors found by the entrepreneur is his private information.

### 3.1.2 Forms of Competition

Competition in the loan market comes in the following two forms:

1. Unskilled lenders can offer the entrepreneur a refinancing of his loan at date 1 after the project has started but before any renegotiation between the entrepreneur and the relationship lender takes place and before the cash flow at date 1 is produced. If the entrepreneur agrees to refinance with one of the competing lenders, the new lender will give the entrepreneur  $R_f$ . The entrepreneur will produce the date-1 cash flow, repay the relationship lender  $R_f$  (the amount of financing the new lender provides), end the first loan contract, and accept the new loan contract with the new lender. As a result, the date-1 repayment to the initial relationship lender is effectively  $R_f$  whether its realized type is patient or impatient, and the new loan constitutes the date-2 portion of the initial loan.
2. Alternatively, after the date-1 cash flow is produced, unskilled lenders can offer to buy the date-2 portion of the initial loan from the relationship lender to obtain the right to collect  $Z_2$ . They get a chance to do so only when the relationship lender is hit with a liquidity shock as the relationship lender will not sell the loan when it is patient. For simplicity, we assume the relationship lender has all the bargaining power when negotiating the loan sale price with any unskilled lender.<sup>11,12</sup>

We examine Bertrand (price) competition in both forms. The entrepreneur refinances his loan with the lender who asks for the lowest repayment at date 2, and the relationship lender sells the loan to the one who submits the highest bid. Note that the entrepreneur can still renegotiate with the new lender before the cash flow at date 2 is produced, as explained

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<sup>10</sup> $N$  and  $\lambda$  may be viewed as the outcomes of banking regulations and banks' marketing efforts. When entry restrictions are relaxed, more banks will emerge and a higher  $N$  leads to a higher degree of competition. Similarly, more aggressive marketing activities increase the probability that a bank is found by the entrepreneur ( $\lambda$ ), which then also results in more intensive competition.

<sup>11</sup>This may be interpreted as an outcome of limited availability of projects that are worth investing in.

<sup>12</sup>Also, note that when loan refinance and loan purchase provide the same expected profit, unskilled lenders prefer to refinance the loan because the relationship lender considers selling the loan at date 1 with only probability  $\theta$  when it receives a liquidity shock and becomes impatient.

before. Also, once receiving a refinancing offer from a competing lender, the entrepreneur allows the incumbent lender to renegotiate with him the date-2 repayment.

### 3.2 Liquidation Technology

There exists a technology that unskilled lenders can purchase at date 1 to *potentially* improve their skill in liquidating the entrepreneur's assets, which then enhances the value they can extract from the loan. We call this the "liquidation technology." It costs  $T > 0$  and produces a new  $\beta$ , called  $\tilde{\beta}'$ , which is a random draw from the support  $[0, 1)$ . A typical example of such a technology is research conducted by lenders to better understand an industry or a project. That is, unskilled lenders acquire knowledge to be more like the relationship lender in terms of liquidation skill.

Let  $\gamma$  be the level of specialization of the project, where  $\gamma \in [0, 1]$ . When  $\gamma = 0$ , it means that the project is least specialized, and when  $\gamma = 1$ , it means that the project is most specialized. Thus,  $\gamma$  is the "type" of the project. The initial  $\beta$  is assumed constant for all  $\gamma$ , implying that the liquidation value is the same for all types of projects to an outsider who possesses no knowledge about the business. However, the distribution of the new  $\tilde{\beta}'$ ,  $F(\tilde{\beta}')$ , is conditional on  $\gamma$ . We assume that for any two values of  $\gamma$ , say  $\gamma_- \in [0, 1]$  and  $\gamma_+ \in [0, 1]$  such that  $\gamma_- < \gamma_+$ , the conditional distribution  $F(\tilde{\beta}'|\gamma_-)$  first-order stochastically dominates  $F(\tilde{\beta}'|\gamma_+)$ . This implies  $\partial F(\tilde{\beta}'|\gamma)/\partial\gamma \geq 0$  and  $\partial E(\tilde{\beta}'|\gamma)/\partial\gamma \leq 0$ . It is reasonable to assume that it is stochastically more difficult to learn to extract value from a more specialized project. To simplify the notation, we define  $\bar{\beta}'(\gamma) \equiv E(\tilde{\beta}'|\gamma)$  and write the expected value of  $\tilde{\beta}'$  as  $\bar{\beta}'$  when  $\gamma$  does not need to be specified.

Suppose that the liquidation technology market is a monopoly. Known at date 0, the cost of acquiring the technology  $T$  is thus endogenously determined by the market, and all unskilled lenders are price-takers. Different projects may have different  $T$ 's as they may have different levels of specialization involved. This technology market has a normal demand curve. That is, the higher the cost of acquiring the liquidation technology, the fewer the unskilled lenders who will purchase it. We denote the demand for the liquidation technology by  $n$  (the number of technology buyers). Clearly,  $n = n(T)$  and  $\frac{\partial n}{\partial T} < 0$ . Having acquired the liquidation technology and a randomly drawn  $\tilde{\beta}'$  at date 1, an unskilled lender can now obtain the maximum of  $\beta X_2$  and  $\tilde{\beta}' X_2$  from liquidating the entrepreneur's assets at date 2. Note that the technology does not guarantee that  $\tilde{\beta}' > \beta$ , and each unskilled lender can get a different realization of  $\tilde{\beta}'$ . All these, including the values of realized  $\tilde{\beta}'$ , are public information. Lastly, we assume that the technology market clears in equilibrium, and no new entry will be attempted unless there is potentially unmet demand for the liquidation technology.

The time line in this setting is same as the one shown in *Figure 1* except for two additions to the model at date 1: (1) at least one competing lender emerges with a probability  $\Lambda$ , and

(2) competing lenders decide how to compete with the incumbent lender (i.e., refinance or purchase the loan) and whether to acquire the liquidation technology. Note that we do not distinguish between the loan being made through direct lending or through insured deposits in this section because the results here do not vary across the two funding choices.

### 3.3 Analysis

Once located by the borrowing entrepreneur at date 1 as a potential lender, an unskilled lender can choose one from three ways to compete for the entrepreneur's business: (1) refinance the loan without acquiring the liquidation technology; (2) acquire the liquidation technology and then refinance the loan; and (3) acquire the liquidation technology and then purchase the loan from the impatient relationship lender.

In this subsection, we first examine lenders' actions and the liquidity of the loan in each of these three cases. Then we provide the conditions under which interbank competition improves loan liquidity. Finally, we examine the likelihood that liquidity will *actually* be created by competition.

#### 3.3.1 Case 1: Loan Refinance without Liquidation Technology

When the date-1 present value of the repayment the entrepreneur can initially commit to paying an unskilled, competing lender at date 2 ( $= \frac{\beta X_2}{R_f}$ ) is higher than the refinancing cost ( $= R_f$ ), this competing lender will prefer to refinance the entrepreneur without acquiring the liquidation technology rather than acquiring the liquidation technology and then refinancing or purchasing the loan. Competition will then drive down the date-2 payment until there is zero profit from lending. Thus, we have:

**Proposition 2** *Suppose  $\beta X_2 \geq R_f^2$ . If a competing lender arrives with probability one, price competition reduces the liquidity of the loan by lowering its maximum market price from  $\frac{\beta X_2}{R_f}$  to  $R_f$  at date 1, but the loan remains liquid. With probability  $\Lambda$  of such competition occurring, the expected liquidity of the loan decreases from  $\frac{\beta X_2}{R_f}$  to  $\Lambda R_f + (1 - \Lambda) \frac{\beta X_2}{R_f}$ .*

This proposition says that the unskilled lenders found by the entrepreneur will compete to refinance the loan without acquiring the liquidation technology when the repayment they can extract from the entrepreneur<sup>13</sup> exceeds their refinancing cost even without the liquidation technology. If there was only one unskilled lender faced with no competition, it would extract a repayment of  $\frac{\beta X_2}{R_f}$  from the borrower. But with multiple competing lenders,<sup>14</sup> the equilibrium outcome of Bertrand competition is  $Z_2 = R_f^2$  as lenders compete the repayment demanded down to their refinancing cost. Anticipating this, the relationship

<sup>13</sup>This is the value they can obtain from liquidating the entrepreneur's assets.

<sup>14</sup>The multiple competing lenders include the incumbent relationship lender and one or more unskilled lenders that the entrepreneur successfully locates.

lender sets  $Z_2$  at  $R_f^2$  in the initial contract. With a repayment of  $R_f^2$  guaranteed at date 2 to either the relationship lender or an unskilled lender, the market value of the loan decreases from  $\frac{\beta X_2}{R_f} (\geq R_f$  in this case) to  $R_f$ , which is the present value of  $R_f^2$  at date 1. That is, interbank competition reduces the value of the loan to a lender by reducing the repayment obligation of the borrower.

Meanwhile, when  $S = \frac{\beta X_2}{R_f} \geq R_f$ , the initial loan is already liquid since  $E_1^I = \max[X_1, S, \frac{X_2}{R_f}] \geq R_f$  ensures that the condition  $L = \min[C_1 + S, E_1^I] \geq R_f$  is satisfied. As the loan liquidity decreases to  $R_f$  with the reduced date-2 payment, the loan remains liquid because  $L \geq R_f$  still holds with  $S = R_f$ .

Note that the results above are derived assuming competition occurs with probability one. Now, consider the general case in which the probability is  $\Lambda \in [0, 1]$  [as defined in (4)] that the entrepreneur will find at least one competing lender, thereby triggering competition. Then, the relationship lender sets the date-2 repayment obligation at  $\frac{\beta X_2}{R_f}$  in the initial contract but is willing to renegotiate it down to  $R_f^2$  if a competing lender arrives. Thus, the *ex ante* expected liquidity of the loan in such a competitive setting is  $\Lambda R_f + (1 - \Lambda) \frac{\beta X_2}{R_f}$ , which is decreasing in the degree of competition.

### 3.3.2 Case 2: Loan Refinance with Liquidation Technology

If  $\beta X_2 < R_f^2$ , an unskilled, competing lender will not refinance the entrepreneur's loan without acquiring the liquidation technology because now its refinancing cost ( $= R_f$ ) exceeds the present value of the expected repayment ( $= \frac{\beta X_2}{R_f}$ ). So, in the absence of the liquidation technology, its only option for obtaining the entrepreneur's business is to buy the loan from an impatient relationship lender. But now, if the expected  $\tilde{\beta}'$  produced by the liquidation technology is such that  $\tilde{\beta}' X_2 \geq R_f^2$ , then buying the technology enables an unskilled lender to improve its liquidation skill "in expectation" to such a level that refinancing the loan becomes feasible. The competing lender will indeed buy the liquidation technology if the price  $T$  is such that its participation constraint can be satisfied. We now examine the condition under which the relationship lender expects an unskilled competitor to purchase the liquidation technology and refinance the entrepreneur's loan.

**Lemma 3** *If  $\beta X_2 < R_f^2$  and  $\tilde{\beta}'(\gamma) X_2 \geq R_f^2$ , then the cost of acquiring the liquidation technology for a project with the level of specialization  $\gamma$  is*

$$T(\gamma) = T_1 \equiv \frac{1}{\lambda N} \left[ R_f - \frac{\beta X_2}{R_f} \right] \quad (5)$$

*in market equilibrium. When  $T(\gamma) = T_1$ , there is a probability  $\Lambda$  that one or more unskilled lenders will first acquire the liquidation technology and then attempt to refinance the entrepreneur's loan.*

Lemma 3 provides the market equilibrium price of the liquidation technology when unskilled lenders expect to increase the liquidation value of the loan above  $R_f^2$  at date

2. That is, the date-1 present value of the expected amount of repayment an unskilled lender can extract after acquiring the liquidation technology ( $= \frac{\bar{\beta}'(\gamma)X_2}{R_f}$ ) now exceeds the refinancing cost ( $= R_f$ ), so this unskilled lender, if approached by the entrepreneur, will compete to refinance the entrepreneur's loan. Nevertheless, competition will ensure that the lender will be able to negotiate a date-2 repayment of only  $R_f^2$  with the refinancing deal. Thus, we see that  $R_f - \frac{\beta X_2}{R_f}$  is the expected gain (valued at date 1) of the repayment negotiated by an unskilled lender that refinances the loan after purchasing the liquidation technology. With a demand of  $n(T)$  homogenous buyers when the liquidation technology is priced at  $T$ , each buyer has a probability  $\frac{1}{n(T)}$  of winning the refinancing deal. Hence, the liquidation technology price should satisfy  $T(\gamma) = \frac{1}{n(T)} \left[ R_f - \frac{\beta X_2}{R_f} \right]$ . Since offering the technology to all potential buyers is a weakly dominant strategy for the monopolist in the liquidation technology market,<sup>15</sup> we obtain  $n(T) = \lambda N$  (the expected number of competing banks found by the entrepreneur) and Lemma 3 follows immediately.

Thus, when  $\bar{\beta}'(\gamma)X_2 \geq R_f^2$  and the entrepreneur finds at least one competing bank, the expected liquidity available to the relationship lenders of both types at date 1 is  $R_f$  (the repayment made through the refinancing funds provided by the new lender), which means that the initial loan becomes liquid in the presence of the liquidation technology. Meanwhile, the expected market value of the loan is also  $R_f$  because this is the date-1 present value of the amount the entrepreneur can commit to paying an unskilled lender at date 2. That is, the liquidity of the loan increases from  $\frac{\beta X_2}{R_f}$  ( $< R_f$  in this case) to  $R_f$ .

### 3.3.3 Case 3: Loan Purchase with Liquidation Technology

If  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma)X_2 < R_f^2$ , an unskilled, competing lender will not refinance the entrepreneur's loan even after acquiring the liquidation technology because its expected liquidation value with the technology still cannot cover the refinancing cost. Instead, it will try to buy the loan from an impatient relationship lender at a price equal to the present value of its liquidation value. Note that the relationship lender sells the loan to the highest bidder. Thus, without knowing how many competing bidders the entrepreneur has found (since it is the entrepreneur's private information), but knowing there is at least another unskilled lender whose probability of being found by the entrepreneur is positive (since  $N \geq 2$  and  $\lambda > 0$ ), this unskilled lender has an incentive to improve its liquidation skill and acquire a higher  $\tilde{\beta}'$  so it can bid higher. If the liquidation technology provides an expected  $\tilde{\beta}' > \beta$ , then the unskilled lender will buy the technology as long as the price  $T$  is such that its participation constraint can be satisfied. We now examine the conditions under which an unskilled lender is expected to purchase the liquidation technology and then buy the loan from the relationship lender.

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<sup>15</sup>This strategy may be strictly dominant if the monopolist wants to deter new entrance to the liquidation technology market.

**Lemma 4** *If  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) X_2 < R_f^2$ , then the cost of acquiring the liquidation technology for a project with the level of specialization  $\gamma$  is*

$$T(\gamma) = T_2 \equiv \frac{\theta}{\lambda N} \left\{ \left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f} \right\} \quad (6)$$

*in market equilibrium. When  $T(\gamma) = T_2$ , there is a probability  $\Lambda$  that one or more unskilled lenders will first acquire the liquidation technology and then attempt to buy the loan from the relationship lender.*

Lemma 4 gives us the equilibrium price of the liquidation technology when the technology provides an expected  $\bar{\beta}'$  that is higher than the initial  $\beta$  but not high enough to make refinancing the entrepreneur's loan a feasible choice for any unskilled lender. With  $\frac{\bar{\beta}'(\gamma)X_2}{R_f}$  being the expected market price of the loan in the presence of the technology, it is easy to see that  $\left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f}$  is the expected gain (valued at date 1) of the unskilled lender that acquires the technology and consequently succeeds in buying the loan from the initial lender. This gain needs to be weighted by  $\theta$ , the probability that the relationship lender is hit with a liquidity shock, because the loan is not for sale when the relationship lender is patient. Similar to our argument in Case 2, the liquidation technology is priced such that all competitors located by the entrepreneur ( $= \lambda N$ ) will buy the technology and each will have a probability  $\frac{1}{\lambda N}$  of winning the loan purchase bid. Hence, the monopolist in the liquidation technology market sets the price at  $\frac{\theta}{\lambda N} \left\{ \left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f} \right\}$ .

Based on the market equilibrium prices of the liquidation technology derived for Cases 2 and 3, we establish the following lemma:

**Lemma 5** *For any  $\gamma$ , there exists an  $T(\gamma) > 0$  if and only if  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) > \beta$ .*

Lemma 5 is a consequence of Lemmas 3 and 4. It says that a positive price for the liquidation technology exists if there is a low liquidation value that can be initially extracted by unskilled lenders ( $\beta X_2 < R_f^2$ ) and an expected gain in liquidation skill after acquiring the technology ( $\bar{\beta}'(\gamma) > \beta$ ). This expected gain may make the unskilled lender's liquidation value go above or remain below the refinancing cost ( $R_f$ ) as indicated by Cases 2 and 3, respectively.

### 3.3.4 Liquidity Creation

We show below that when the liquidation technology is available at a positive price,  $T(\gamma) > 0$ , competition in the loan market will increase the expected liquidity of the loan and reduce the liquidity risks for both the incumbent relationship lender and the borrowing entrepreneur. Such an effect is stronger with greater competition.

In this setting, the probability that Bertrand competition occurs among banks in the loan market is  $\Lambda$ , as defined in Equation (4). Let  $S'$  denote the loan sale price when

competition *actually* occurs.<sup>16</sup> Thus, the expected loan sale price is

$$S^* \equiv \Lambda S' + (1 - \Lambda) S, \quad (7)$$

where  $S' = \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right]$  and  $S = \frac{\beta X_2}{R_f}$ .

Equation (7) can be rearranged as

$$S^* = S + \Lambda \left\{ \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right] - \frac{\beta X_2}{R_f} \right\}. \quad (8)$$

With  $\Lambda > 0$  and both  $\bar{\beta}'(\gamma) > \beta$  and  $R_f > \frac{\beta X_2}{R_f}$  implied by the positive  $T(\gamma)$  (Lemma 5), it is clear that  $S^* > S$ . Therefore, we conclude that loan liquidity is expected to increase when there is competition in the loan market.

The higher expected market price of the loan attributable to interbank competition reduces the liquidity risk for the relationship lender as it can expect to sell the loan at a price that is closer to the full value of the loan. The liquidity risk faced by the borrowing entrepreneur also decreases from two perspectives. First, he pays a lower liquidity premium [Equation (1)]. This is because the lender expects higher proceeds from a loan sale ( $S$ ), which then result in a higher  $V_1^*$  ( $= C_1 + S$ ), when it suffers a liquidity shock and becomes impatient; Equation (1) shows that this reduces the liquidity premium,  $\mathcal{LP}$ , for the entrepreneur. Second, the conditions under which the relationship lender is willing to make an illiquid loan with a plan to sell it contingent upon receiving a liquidity shock at date 1 (i.e., Conditions 1 and 2) become less stringent as the expected market price of the loan increases. This leads to an increase in the probability that the entrepreneur will pay the smallest possible liquidity premium.

Now, we write the expected gain in loan liquidity as:

$$G \equiv S^* - S = \Lambda \left\{ \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right] - \frac{\beta X_2}{R_f} \right\}. \quad (9)$$

From the expression (9), we see that  $G$  is increasing in  $\Lambda$ . That is, the higher the probability that Bertrand competition occurs in the loan market ( $\Lambda$ ), the greater is the expected gain in loan liquidity ( $G$ ). We know that  $\frac{\partial \Lambda}{\partial N} > 0$  and  $\frac{\partial \Lambda}{\partial \lambda} > 0$ . Thus, the liquidity-creation effect of interbank competition increases in the competition intensity measures  $N$  and  $\lambda$ .

We now reexamine the condition for a loan to be liquid in the competitive setting. In the absence of interbank competition, a loan is liquid if  $L = \min [C_1 + S, E_1^I] \geq R_f$ . With competition, the condition becomes

$$L^* \equiv \min \left\{ C_1 + S^*, \max \left[ X_1, S^*, \frac{X_2}{R_l} \right] \right\} \geq R_f, \quad (10)$$

which can be satisfied if the price for acquiring the liquidation technology and/or the degree of competition permits it. To simplify the discussion of various scenarios, we assume:

<sup>16</sup>Recall that (1)  $S' = R_f$  if  $\frac{\bar{\beta}'(\gamma) X_2}{R_f} \geq R_f$  (Case 2) and (2)  $S' = \frac{\bar{\beta}'(\gamma) X_2}{R_f}$  if  $\frac{\bar{\beta}'(\gamma) X_2}{R_f} < R_f$  (Case 3).

- Assumption 7:  $C_1 + R_f \leq \max \left[ X_1, \frac{X_2}{R_l} \right]$ .

Since  $S^* \leq S' \leq R_f$ , Assumption 7 says that the maximum liquidity available at date 1 for the entrepreneur to pay the relationship lender when price competition actually occurs ( $C_1 + S^*$ ) is always within the range of repayments that the entrepreneur can commit to making. That is, the entrepreneur's repayment commitment is renegotiation-proof. With this assumption, the condition specified in (10) effectively becomes

$$L^* = C_1 + S^* \geq R_f. \quad (11)$$

The following proposition formally states the results discussed above and provides the conditions for (11) to hold:

**Proposition 3** *For any project type  $\gamma$ , given the existence of a liquidation technology that can be purchased for a price  $T(\gamma) > 0$ , we have:*

- i. Interbank competition is expected to improve the liquidity (i.e., the maximum market price) of an illiquid loan for a project of type  $\gamma$  and hence lower the liquidity risks faced by both the relationship lender and the entrepreneur;*
- ii. The liquidity created by interbank competition increases with the number of competing banks ( $N$ ) and the probability of the entrepreneur successfully locating each bank ( $\lambda$ );*
- iii. Interbank competition transforms an illiquid loan into a liquid loan if  $T(\gamma)$  and/or  $\Lambda$  satisfies either of the following conditions:*

$$(a) \quad T(\gamma) = T_1 \quad \text{and} \quad \Lambda \geq \underline{\Lambda} \equiv 1 - \frac{C_1}{R_f - \beta X_2 / R_f},$$

$$(b) \quad T_1 > T(\gamma) \geq \underline{T}_2 \equiv \frac{\theta(R_f - C_1 - \beta X_2 / R_f)}{\Lambda \cdot \lambda N}.$$

Proposition 3 tells us that loan liquidity is improved by interbank competition as long as the liquidation technology is available at a positive price. The expected gain in liquidity increases with the degree of competition. Moreover, an illiquid loan becomes liquid due to competition if the liquidation technology price and/or the degree of competition is sufficiently high. If the liquidation technology is so good that acquiring it makes refinancing the entrepreneur's loan feasible for an unskilled lender, then the level of competition needs to be sufficiently high in order to transform the illiquid loan into a liquid loan. Alternatively, if the liquidation technology is not that good, a sufficiently high price for the technology is needed in order for the transformation to take place.

To understand the intuition, we begin by noting that since  $\beta X_2 < R_f^2$ , unskilled lenders find it unprofitable to refinance the borrower without acquiring the liquidation technology (something that they can do when  $\beta X_2 \geq R_f^2$ ). The existence of a market for the liquidation technology with a positive price means that an unskilled lender purchasing the technology

can expect to improve its liquidation skill, which in turn would allow it to extract a higher repayment from the borrower. This effectively raises the market price of the loan and thus improves its liquidity. Since such a gain does not occur unless the entrepreneur finds at least one competing unskilled lender, the expected increase in the liquidity of the loan depends on the level of competition in the loan market. This competition level is captured by two parameters: the number of competing lenders ( $N$ ) and the probability that a lender will be located by the entrepreneur ( $\lambda$ ). The higher are  $N$  and  $\lambda$ , the more likely it is that the initial lender will *actually* face competition.

The conditions specified in Proposition 3 become easier to satisfy if the project being financed by the loan has the following characteristics:

- $C_1$  is relatively high: When the project can produce a higher cash flow in the interim period, the loan needs only a smaller increase in its expected market price to become liquid since (11) says that we need  $S^* \geq R_f - C_1$ , which is decreasing in  $C_1$ .
- $X_2$  is relatively high: In this case, first, the loan does not need too large an increase in an unskilled lender's liquidation skill ( $\beta$ ) since the distance from being liquid ( $R_f - \beta X_2$ ) is small. Second, even a small increase from  $\beta$  to  $\bar{\beta}'(\gamma)$  results in a large gain in the lender's liquidation value.
- $\gamma$  is relatively small: Due to the assumption of first-order stochastic dominance,  $\partial \bar{\beta}'(\gamma) / \partial \gamma \leq 0$ . So, a less specialized project with a smaller  $\gamma$  can achieve a relatively higher  $\bar{\beta}'(\gamma)$ , which then makes the liquidation technology price  $T(\gamma)$  higher.

That is, competition is more likely to enhance liquidity if the project being financed by the loan has a higher interim-period cash flow ( $C_1$ ), a higher maximum liquidation value ( $X_2$ ), and/or a lower level of specialization ( $\gamma$ ).

Table 2 summarizes for each case the values of  $\beta X_2$  and  $\bar{\beta}'(\gamma) X_2$ , competitors' actions, the equilibrium price of the liquidation technology, the effect of competition on loan liquidity, and the condition for transforming an illiquid loan to a liquid loan.

Table 2. Summary of Cases 1-3

Case	1	2	3
$\beta X_2$	$\geq R_f^2$	$< R_f^2$	$< R_f^2$
$\bar{\beta}'(\gamma) X_2$	–	$\geq R_f^2$	$< R_f^2$
Competitors' actions:			
Acquire the liquidation technology?	No	Yes	Yes
Loan refinance or loan purchase?	Refinance	Refinance	Purchase
Price of the liquidation technology	None	$T(\gamma) = T_1$	$T(\gamma) = T_2$
Effect of competition on loan liquidity	Reduce	Improve	Improve
Condition for having a liquid loan	Always liquid	$\Lambda \geq \underline{\Lambda}$	$T(\gamma) \geq \underline{T_2}$

### 3.3.5 Realization of Liquidity Creation

Proposition 3 is based on the relationship lender's expectation. How likely is it that liquidity will actually be created by competition? That is, how likely is it that at least one realized  $\tilde{\beta}'$  will be higher than the initial  $\beta$ ?

Suppose that  $n$  unskilled lenders acquire  $\tilde{\beta}'$  independently. Then, we get

$$\Pr\left(\text{having at least one } \tilde{\beta}' > \beta\right) = 1 - F(\beta|\gamma)^n. \quad (12)$$

This probability is decreasing in  $F(\beta|\gamma)$  and increasing in  $n$ . Thus, we need a low probability that the randomly drawn  $\tilde{\beta}'$  is less than the initial  $\beta$  and/or a large number of unskilled lenders who invest in the liquidation technology.

What types of projects have a low probability of  $\tilde{\beta}' < \beta$ ? The assumption of first-order stochastic dominance implies that  $\partial F(\tilde{\beta}'|\gamma)/\partial\gamma \geq 0$ . That is,  $F(\beta|\gamma)$  is decreasing in the level of specialization  $\gamma$  of the entrepreneur's project.

When will a large number of unskilled lenders acquire better liquidation skills? From the derivation of Lemmas 3 and 4, we know that in equilibrium the liquidation technology price is set such that all unskilled lenders found by the entrepreneur acquire the technology. That is,  $n (= \lambda N)$  is increasing in the degree of competition in the loan market.

Thus, we conclude that liquidity creation through competition is more likely to be *realized* with a less specialized project financed by the loan and/or a higher degree of competition.

## 4 Credit Risk

Sections 2 and 3 analyze liquidity risk and the impact of interbank competition on liquidity risk in a simple, one-state world that is free of credit risk. We now use the results established so far as building blocks to understand these issues when loan returns are stochastic and credit risk is present. We show that: (1) banks may be induced to take on additional credit risk to manage liquidity risk, and (2) interbank competition not only reduces liquidity risk under some conditions but also makes credit risk comonotonic with liquidity risk, so banks are not tempted to take additional credit risk to mitigate liquidity risk. One important implication of the latter, though, is that high credit risk and high liquidity risk may coexist for banks whose credit risk exceeds a certain threshold.

### 4.1 Risk: State of the World

We introduce credit risk by assuming that the project financed by the loan has a stochastic payoff. This permits us to examine the interaction between credit and liquidity risks.

Suppose that project payoffs and asset values depend on some macroeconomic factors over which the entrepreneur has no control, such as inflation, unemployment rate, etc.

These factors can be described by the state of the world,  $\tilde{\alpha}$ , which is revealed to all agents at date 1 after the investment is made but before the date-1 cash flow is produced. Let  $\tilde{\alpha}$  be a random draw from a uniform distribution over the support of  $[1 - \delta, 1 + \delta]$ , where  $\delta \in \left(1 - \frac{r_f + \eta}{C_1 + S}, 1\right]$ . Thus,  $\tilde{\alpha}$  has an expectation of 1 and a density function of  $\frac{1}{2\delta}$ . The range of  $\delta$  is set such that there is default risk for the lending bank.

Conditional on the common factor  $\tilde{\alpha}$ , project payoffs and asset values are now random variables. Suppose that  $\tilde{C}_1 = \tilde{\alpha}C_1$ ,  $\tilde{C}_2 = \tilde{\alpha}C_2$ ,  $\tilde{X}_1 = \tilde{\alpha}X_1$ , and  $\tilde{X}_2 = \tilde{\alpha}X_2$ . We then have the expected cash flows of  $C_1$  and  $C_2$  and liquidation values of  $X_1$  and  $X_2$  at date 1 and date 2, respectively. Since  $\tilde{S} = \frac{\beta\tilde{X}_2}{R_f}$ , the expected maximum selling price of the loan is  $\frac{\beta X_2}{R_f}$ , i.e.,  $S$ .<sup>17</sup> In summary, the expected values of the random variables in this case are the same as the actual values in the certainty case described in Section 2. Meanwhile,  $\delta$ , which measures the dispersion of  $\tilde{\alpha}$ , can be viewed as a measure of the credit risk of the loan: the larger the value of  $\delta$ , the higher the default probability of the loan.<sup>18</sup>

The loan contract is made contingent on the realization of  $\tilde{\alpha}$ . That is, it specifies  $Z_t$  as the expected payment at date  $t$ ; upon the realization of  $\tilde{\alpha}$ , the actual payment is  $\tilde{\alpha}Z_t$ .

Let lending continue to be provided under financial intermediation with complete deposit insurance. Assumptions 1-6 remain as before. Based on Assumptions 1-3, we know that on average the project can be financed if the relationship lender suffers no liquidity shock. Assumptions 4-5 help us focus on selected cases based on project payoffs and asset values instead of all possible scenarios. Assumption 6 says that on average the bank does not need to sell or liquidate the loan to keep itself solvent. Also, we continue to assume that the bank is financially constrained at date 1 in the sense that it is unable to raise additional funds at that time. We exclude bank competition for now.

The time line in this setting is also similar to the one shown in *Figure 1* with the additions of both the financial intermediation features described in Section 2.2 and the realization of the random state of the world at date 1.

## 4.2 Lender's Choices and Payoffs

With uncertain cash flows and asset values, the relationship lender's action at date 1 depends on not only its realized liquidity need but also the realized state of the world. Let  $a_1 \equiv \frac{r_f + \eta}{C_1 + S}$  and  $a_2 \equiv \frac{r_f + \eta}{C_1}$ . We can write the range of  $\delta$  as  $\delta \in (1 - a_1, 1]$ . Then,

1. If  $\tilde{\alpha} < a_1$ , the lenders of both types will be shut down at date 1 because they cannot make the promised interest payments to the depositors and pay back the funds that

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<sup>17</sup>Note that  $S$  is indeed the expected liquidity of the loan to the relationship lender in the absence of default risk. In our case, however, there are states in which the lender fails to make the promised payments and will be closed, and hence the loan is worth zero even though it can still be sold for  $\frac{\beta\tilde{X}_2}{R_f}$  in the secondary market. As a result, the expected liquidity of the loan to the lender is somewhere below  $S$ .

<sup>18</sup>The probability that the loan will default can be written as  $\Pr(\text{Default}) = \left[\frac{r_f + \eta}{C_1 + S} - (1 - \delta)\right] / 2\delta$ . We take the first derivative of this probability with regard to  $\delta$  and get  $\partial \Pr(\text{Default}) / \partial \delta = \left(1 - \frac{r_f + \eta}{C_1 + S}\right) / 2\delta^2 > 0$ .

are withdrawn at date 1 no matter what actions they take.

2. If  $a_1 \leq \tilde{\alpha} < a_2$ , the lenders of both types will sell the loan in order to make the promised interest payment and satisfy demanded withdrawals at date 1.
3. If  $\tilde{\alpha} \geq a_2$ , the impatient lender will either sell, keep, or liquidate the loan according to the contract, while the patient lender will always keep the loan.

The payoffs to the relationship lender differ across the three cases described above. Since the value of  $\delta$  admits all three cases as possibilities, the total expected payoff is the sum of the expected payoff in each situation weighted by its probability of occurrence.

No matter how small the realized  $\tilde{\alpha}$  is, the entrepreneur will produce the cash flows (as long as the loan contract is renegotiation-proof), viewing the initial investment as a sunk cost. Using the same arguments as before, he will pay no more than  $\tilde{\alpha}X_2$  at date 2, no more than a date-1 present value of  $\tilde{E}_1^P \equiv \tilde{\alpha}E_1^P$  to the patient lender (if  $\tilde{\alpha}$  is not sufficiently small to force the patient lender to sell the loan) and no more than  $\tilde{E}_1^I \equiv \tilde{\alpha}E_1^I$  to the impatient lender over two dates. Meanwhile, the maximum liquidity he can offer the relationship lender at date 1 is  $\tilde{\alpha}(C_1 + S)$  if the loan is sold.

Nevertheless, the actual payoffs to the relationship lender may fall short of what the entrepreneur offers. In states with low  $\tilde{\alpha}$ , even the patient relationship lender operating in a financial-intermediation setting may default or have to sell the loan.<sup>19</sup> In Scenario 1 where  $\tilde{\alpha} < a_1$ , the liquidity of the loan is zero and the lenders of both types face a payoff of zero. In Scenario 2 where  $a_1 \leq \tilde{\alpha} < a_2$ , the liquidity of the loan is  $\tilde{\alpha}S$  and the lender of both types can extract as much as  $\tilde{\alpha}(C_1 + S)$ . Finally, in Scenario 3 where  $\tilde{\alpha} \geq a_2$ , the liquidity of the loan is again  $\tilde{\alpha}S$  and the largest renegotiation-proof amounts (valued at date 1) for the patient and impatient lenders are  $\tilde{E}_1^P$  and  $\tilde{E}_1^I$ , respectively.

### 4.3 Expected Liquidity Premium

We now examine the expected liquidity premium valued at date 2. Similar to the arguments before, we have two relationships:  $E[\widetilde{\mathcal{LP}}] = E[\widetilde{V}] + \theta E[\widetilde{V}_1^*] R_f - R_f^2$  and  $E[\widetilde{V}] + \theta E[\widetilde{V}_1^*] R_l = \underline{U}$ . Then, the expected liquidity premium can be computed as:

$$E[\widetilde{\mathcal{LP}}] = \theta(R_l - R_f) \left( R_f - E[\widetilde{V}_1^*] \right). \quad (13)$$

Thus, if the expected cash flow to the impatient lender at date 1 is equal to  $R_f$ , the entrepreneur obtains financing with an expected liquidity premium of zero. We call a risky loan *liquid* if it can be made at a zero expected liquidity premium.

<sup>19</sup>This is because: (1) an interim-period interest is promised in the deposit contract, and (2) depositors may receive a liquidity shock and demand immediate withdrawals.

Based on the discussion about the lender's choices and payoffs, the expected cash flow to the impatient lender at date 1 can be computed as follows:

$$E \left[ \tilde{V}_1^* \right] = \int_{1-\delta}^{a_1} 0 \cdot \frac{1}{2\delta} d\tilde{\alpha} + \int_{a_1}^{1+\delta} \min \left[ \tilde{\alpha} (C_1 + S), \tilde{\alpha} E_1^I \right] \cdot \frac{1}{2\delta} d\tilde{\alpha}. \quad (14)$$

Let  $A_1 \equiv \frac{(1+\delta)^2 - a_1^2}{4\delta}$ ,  $A_2 \equiv \frac{(1+\delta)^2 - a_2^2}{4\delta}$ , and  $K_1 \equiv A_1 - A_2 = \frac{a_2^2 - a_1^2}{4\delta}$ . Then, we can write (14) as:

$$E \left[ \tilde{V}_1^* \right] = A_1 \cdot \min \left[ C_1 + S, E_1^I \right]. \quad (15)$$

Thus, the condition for this risky loan to be liquid is

$$L = \min \left[ C_1 + S, E_1^I \right] \geq \underline{L}_1 \equiv A_1^{-1} R_f. \quad (16)$$

With  $\delta \in (1 - a_1, 1]$ , it is easy to verify that  $A_1^{-1} > 1$ . Assumption 4 says that  $L = C_1 + S < R_f$ . Thus, this risky loan is illiquid since  $L < R_f < A_1^{-1} R_f$ .

We now move on to analyze the liquidity risk embedded in this illiquid, risky loan. In the following lemma, we discuss the condition for the loan to be sold by the impatient lender at date 1 when  $\tilde{\alpha} \geq a_2$ .

**Lemma 6** *Suppose Assumptions 1-6 hold. Further: (1) cash flows produced by the project and its liquidation values are risky as described above, and (2) lending is provided through financial intermediation with complete deposit insurance. Then, if the following condition holds,*

$$E_1^P \geq \underline{E}_3 \equiv \left\{ R_f + \frac{\theta}{1-\theta} \frac{R_l}{R_f} [R_f - A_1 (C_1 + S)] - K_1 (C_1 + S) \right\} A_2^{-1} \text{ and}$$

$$C_1 + S \geq \underline{CS}_3 \equiv \frac{E_1^P R_f}{R_l} + (r_f + \eta) \left( 1 - \frac{R_f}{R_l} \right) \left( \frac{2}{1 + \delta + a_2} \right), \quad (\text{Condition 3})$$

the entrepreneur will be financed at date 0 and the relationship lender will sell the loan at date 1 when  $\tilde{\alpha} < a_2$ , and the impatient lender will do so even when  $\tilde{\alpha} \geq a_2$ .

Like Lemmas 1 and 2, Lemma 6 provides restrictions on the project's cash payoffs and asset values. We now proceed to analyze the interaction between liquidity risk and credit risk by examining how an increase in  $\delta$  affects the stringency of these restrictions.

#### 4.4 The Interaction Between Liquidity Risk and Credit Risk

We examine how the liquidity conditions vary with the credit risk measure  $\delta$ .

First, we compare the financing condition for a risky project (Condition 3) to that for a riskless project (Condition 2). It can be shown that if  $C_1 + S > R_f (1 - A_2) K_1^{-1}$ , then  $\underline{E}_2 > \underline{E}_3$ . Meanwhile, if  $\delta > \underline{\delta}_1 \equiv 1 - a_2$ , it is always true that  $\underline{CS}_2 > \underline{CS}_3$ .<sup>20</sup> That is, a

<sup>20</sup>The condition  $\delta > \underline{\delta}_1$  always holds in our setting since  $\delta > 1 - a_1$  and  $a_1 < a_2$  by definition.

project with risky payoffs may be financed with a lower liquidity premium than a riskless project whose certain payoff equals the expected value of the risky project's payoff.

Next, we take the first derivative of  $\underline{L}_1$ , the lower bound of  $\min [C_1 + S, E_1^I]$  that makes a risky loan liquid, with respect to  $\delta$ :

$$\frac{\partial \underline{L}_1}{\partial \delta} = \frac{4R_f}{\left[(1 + \delta)^2 - a_1^2\right]^2} (1 - a_1^2 - \delta^2). \quad (17)$$

We see that for all  $\delta > \underline{\delta}_2 \equiv \sqrt{1 - a_1^2}$ ,  $\frac{\partial \underline{L}_1}{\partial \delta} < 0$  and hence  $\underline{L}_1$  decreases in  $\delta$ . Since  $\delta$  measures the credit risk of the loan, this result means that *ceteris paribus*, a loan with higher credit risk may be liquid while a loan with lower credit risk is not. In this case, the entrepreneur with the lower-credit-risk project faces a higher liquidity premium and may possibly be denied financing.

Finally, we examine Condition 3. We rewrite  $\underline{E}_3$  as

$$\underline{E}_3 \equiv \Gamma \cdot A_2^{-1} = \left\{ R_f + \frac{\theta}{1 - \theta} \frac{R_l}{R_f} [R_f - A_1 (C_1 + S)] - K_1 (C_1 + S) \right\} A_2^{-1}. \quad (18)$$

Let us take the first derivative of both the denominator and the numerator of  $\underline{E}_3$  with respect to  $\delta$ :

$$\frac{\partial \Gamma}{\partial \delta} = -\frac{C_1 + S}{4\delta^2} \left[ \frac{\theta}{1 - \theta} \frac{R_l}{R_f} (\delta^2 - 1 + a_1^2) - (a_2^2 - a_1^2) \right]; \quad (19)$$

$$\frac{\partial A_2}{\partial \delta} = \frac{\delta^2 - 1 + a_2^2}{4\delta^2}. \quad (20)$$

It can be shown that if  $\delta > \underline{\delta}_3 \equiv \sqrt{\frac{1 - \theta}{\theta} \frac{R_l}{R_f} (a_2^2 - a_1^2) + (1 - a_1^2)}$ ,  $\frac{\partial \Gamma}{\partial \delta} < 0$  and  $\frac{\partial A_2}{\partial \delta} > 0$ . Then, as  $\delta$  increases,  $\underline{E}_3$  decreases. Meanwhile,  $\underline{CS}_3$  decreases in  $\delta$  when  $E_1^P$  is held constant. Thus, we say that a project with higher credit risk may obtain financing with a lower liquidity premium than a project with lower credit risk even if the two projects have the same expected payoffs.

We now summarize these results in the following proposition.

**Proposition 4** *When project payoffs and asset values are uncertain, a loan with higher credit risk may have lower liquidity risk in the following sense:*

- i. A risky project may be able to obtain financing when a riskless project with a payoff equal to the expected value of the payoff on the risky project does not, and the risky project may have a lower liquidity premium than the riskless project.*
- ii. Among two risky projects with equal expected payoffs, the riskier project may obtain financing when the less risky project does not, and the riskier project may have a lower liquidity premium than the less risky project.*

Proposition 4 states that liquidity risk does *not* necessarily increase with the credit risk of the loan measured by  $\delta$ . The reason is the put option effect created by deposit insurance. The payoff distribution of a loan with higher credit risk has fatter tails than the payoff distribution of a lower-credit-risk loan, with higher probabilities of both very high and very low payoffs. Thus, a loan with higher credit risk may provide a higher *conditional* expected payoff to the bank since deposit insurance truncates payoffs below the promised interest payment plus the expected level of withdrawals ( $r_f + \eta$ ), without affecting the payoffs in the higher states. Higher credit risk may consequently lead to a higher resale price for the loan, implying higher liquidity or lower liquidity risk. In other words, liquidity risk and credit risk may not be comonotonic.

This result establishes that a bank's liquidity need, arising either from the possibility of being hit with a liquidity shock or because of the need to make payments promised to depositors, could induce it to take on higher credit risk. The bank's need for liquidity, along with the inability to sell the loan at its full value, results in liquidity risk. Although the cost associated with this liquidity risk is borne *ex ante* by the entrepreneur, it eventually becomes the bank's problem if it takes excessive credit risk to manage its liquidity risk.<sup>21</sup>

#### 4.5 The Role of Interbank Competition

We now examine how competition affects both liquidity risk and credit risk. The time line in this more complex setting also follows *Figure 1* with the additions of (1) the financial intermediation features described in Section 2.2, (2) the realization of the random state of the world that gives rise to banks' default risk, and (3) the emergence of interbank competition in the loan market.

Suppose that there exists a liquidation technology that produces an expected  $\bar{\beta}'$  that is higher than the initial  $\beta$ , i.e.,  $\bar{\beta}' > \beta$ . To simplify, we continue to maintain Assumption 7. In addition, we assume:

- Assumption 8:  $\beta X_2 < \bar{\beta}' X_2 < R_f^2$ .
- Assumption 9:  $(1 - \delta) \left( C_1 + \frac{\bar{\beta}' X_2}{R_f} \right) < r_f + \eta$ .
- Assumption 10:  $(1 + \delta) \beta X_2 > R_f^2$ .

Assumption 7 ensures that the entrepreneur can credibly commit to paying a competing lender  $\tilde{\alpha} C_1 + \tilde{S}'$ , where  $\tilde{S}'$  is the maximum selling price of the loan after an unskilled lender acquires the liquidation technology. Assumption 8 indicates that the expected liquidation value of an unskilled lender either before or after it acquires the liquidation technology is below the refinancing cost. Assumption 9 makes bank default possible even with a

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<sup>21</sup>Although it is outside the scope of our analysis, this excessive credit risk can also lower borrowers' welfare. See Besanko and Thakor (1993).

liquidation technology. Assumption 10 permits the existence of some (high) states within the range of  $\tilde{\alpha}$  that an unskilled lender's liquidation value even without the liquidation technology is higher than the refinancing cost.

As in the certainty case, at date 1 an unskilled lender that has been found by the entrepreneur chooses whether to acquire the liquidation technology and whether to refinance the entrepreneur's loan or buy the loan from the relationship lender. In contrast to the previous setting, now the project produces risky cash flows depending on the state of the world,  $\tilde{\alpha}$ , which is observed by all after the investment is made. Hence, this competing unskilled lender decides what to do after observing  $\tilde{\alpha}$ .

Recall that  $a_1 \equiv \frac{r_f + \eta}{C_1 + S}$  and  $a_2 \equiv \frac{r_f + \eta}{C_1}$ . We further define  $a_0 \equiv \frac{r_f + \eta}{C_1 + \beta' X_2 / R_f}$  and  $a_3 \equiv \frac{R_f^2}{\beta' X_2}$ . Applying the results derived in Section 3 to different scenarios depending on the realized  $\tilde{\alpha}$ , we find that when competition *actually* occurs, the relationship lender's payoff is zero if  $\tilde{\alpha} < a_0$ , whereas its expected present value is  $R_f$  at date 1 if  $\tilde{\alpha} \geq a_3$ .

A positive liquidation technology price,  $\tilde{T}$ , exists if  $a_0 \leq \tilde{\alpha} < \frac{R_f}{S}$ . A price  $T$  is observed before the state of the world is realized, and it is the price of the liquidation technology assuming all parameters are at their expected values. That is,  $T = T_2$ .<sup>22</sup>

#### 4.5.1 Liquidity Creation

With probability  $\Lambda$  Bertrand competition occurs in the loan market, so the expected liquidity of the loan across all possible states is

$$E(\tilde{S}^*) = \Lambda E(\tilde{S}') + (1 - \Lambda) E(\tilde{S}), \quad (21)$$

where

$$E(\tilde{S}') = \int_{a_0}^{a_3} \tilde{\alpha} S' \cdot \frac{1}{2\delta} d\tilde{\alpha} + \int_{a_3}^{1+\delta} R_f \cdot \frac{1}{2\delta} d\tilde{\alpha}, \quad (22)$$

$$E(\tilde{S}) = \int_{a_1}^{1+\delta} \tilde{\alpha} S \cdot \frac{1}{2\delta} d\tilde{\alpha}. \quad (23)$$

$S'$  is the expected sale price of the loan when competition *actually* occurs and  $\tilde{\alpha} = 1$ , and we know that  $S' = \frac{\beta' X_2}{R_f}$ . Then, the expected gain (or loss) in loan liquidity is

$$E(\tilde{G}) = E(\tilde{S}^*) - E(\tilde{S}) = \Lambda \left[ E(\tilde{S}') - E(\tilde{S}) \right]. \quad (24)$$

We say that competition creates liquidity if  $E(\tilde{G}) > 0$ .<sup>23</sup>

*Figure 2* helps visualize the effect of interbank competition on the liquidity of the loan. In both *Panels A* and *B* of *Figure 2*, we have on the  $x$ -axis the value of the loan to an outside, unskilled lender before the liquidation technology is acquired (i.e.,  $\tilde{\alpha}S$ ) and on the

<sup>22</sup>The actual price  $\tilde{T}(\gamma)$  is weakly increasing in  $\tilde{\alpha}$ : it is first strictly increasing in  $\tilde{\alpha}$  if  $\tilde{\alpha} \in [a_0, a_3]$ , then stays constant at  $T_1$  for all  $\tilde{\alpha} \in [a_3, \frac{R_f}{S}]$ .

<sup>23</sup>That is, competition increases the expected market price of the loan, not necessarily makes an initially illiquid loan liquid.

$y$ -axis the liquidity of the loan to the relationship lender. The solid red line represents loan liquidity in the absence of interbank competition, which is zero if  $\tilde{\alpha} < a_1$  and has a slope of one if  $\tilde{\alpha} \geq a_1$ . The dashed green line represents loan liquidity in the presence of interbank competition, which is zero if  $\tilde{\alpha} < a_0$ , has a slope of  $\frac{\beta'}{\beta}$  if  $a_0 \leq \tilde{\alpha} < a_3$ , and stays constant at  $R_f$  once  $\tilde{\alpha} > a_3$ . There are two sources of liquidity gain due to competition: (1) the relationship lender is saved from being shut down in the lower states where  $a_0 \leq \tilde{\alpha} < a_1$ , and (2) the loan sale price increases in the middle states where  $a_1 \leq \tilde{\alpha} < \frac{R_f}{S}$ . Meanwhile, competition also causes loss in loan liquidity by reducing the market value of the loan to  $R_f$  in the higher states where  $\tilde{\alpha} \geq \frac{R_f}{S}$ . In *Figure 2*, the shaded green area represents liquidity gain, whereas the shaded red area represents liquidity loss. Whether competition improves or worsens loan liquidity (i.e., whether the green area is greater or smaller than the red area) depends on how much credit risk is embedded in the project that is being financed by the loan. *Panels A and B of Figure 2* differ in their sizes of credit risk  $\delta$ . *Panel A* shows a project with relatively low credit risk, whereas *Panel B* shows a project with relatively high credit risk. It can be seen that the total gain in loan liquidity represented by the green area is fixed, whereas the total loss represented by the red area is increasing in  $\delta$ . We show these results more rigorously in the following analysis.

Recall that  $a_0 \equiv \frac{r_f + \eta}{C_1 + \beta' X_2 / R_f}$ ,  $a_1 \equiv \frac{r_f + \eta}{C_1 + S}$ ,  $a_2 \equiv \frac{r_f + \eta}{C_1}$ , and  $a_3 \equiv \frac{R_f^2}{\beta' X_2}$ . Also, we've defined  $A_1 \equiv \frac{(1+\delta)^2 - a_1^2}{4\delta}$ ,  $A_2 \equiv \frac{(1+\delta)^2 - a_2^2}{4\delta}$ , and  $K_1 \equiv \frac{a_2^2 - a_1^2}{4\delta}$ . Now, let  $A_0 \equiv \frac{(1+\delta)^2 - a_0^2}{4\delta}$ ,  $A_3 \equiv \frac{(1+\delta)^2 - a_3^2}{4\delta}$ ,  $K_0 \equiv A_0 - A_3 = \frac{a_3^2 - a_0^2}{4\delta}$ ,  $K_2 \equiv A_0 - A_2 = \frac{a_2^2 - a_0^2}{4\delta}$ , and  $K_3 \equiv A_2 - A_3 = \frac{a_3^2 - a_2^2}{4\delta}$ . Then, we obtain:

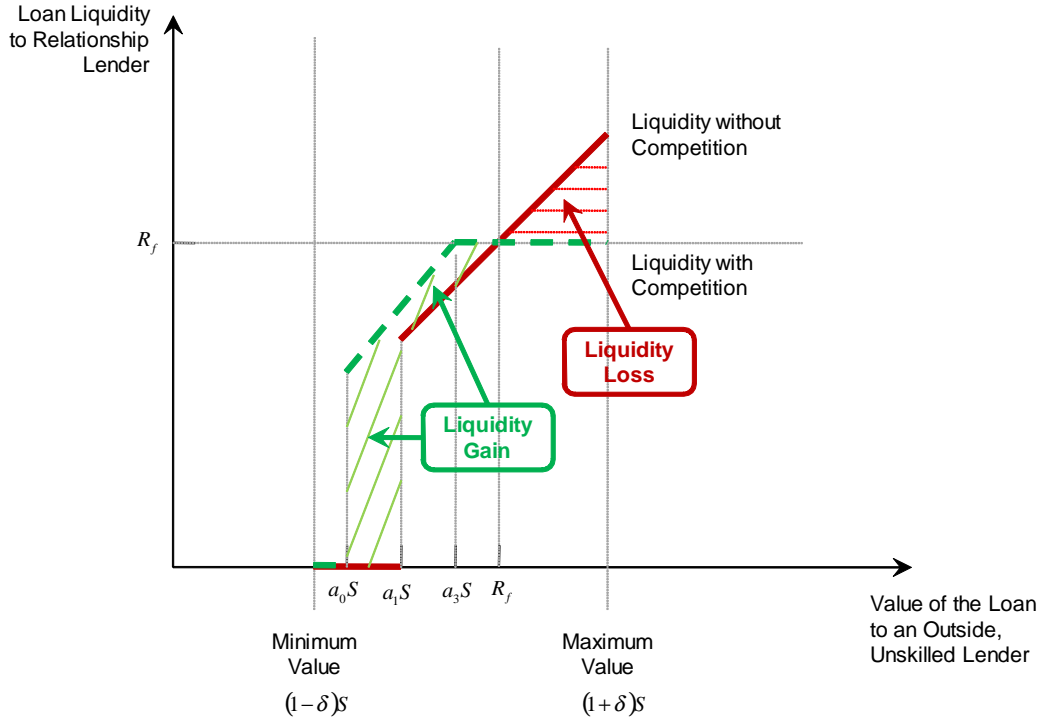
$$E(\tilde{G}) = \Lambda \left[ K_0 \cdot S' + \left( \frac{1 + \delta - a_3}{2\delta} \right) R_f - A_1 \cdot S \right]. \quad (25)$$

The following proposition provides the condition for competition to create liquidity and how the expected gain in loan liquidity varies with  $T$ ,  $\delta$ , and  $\Lambda$ :

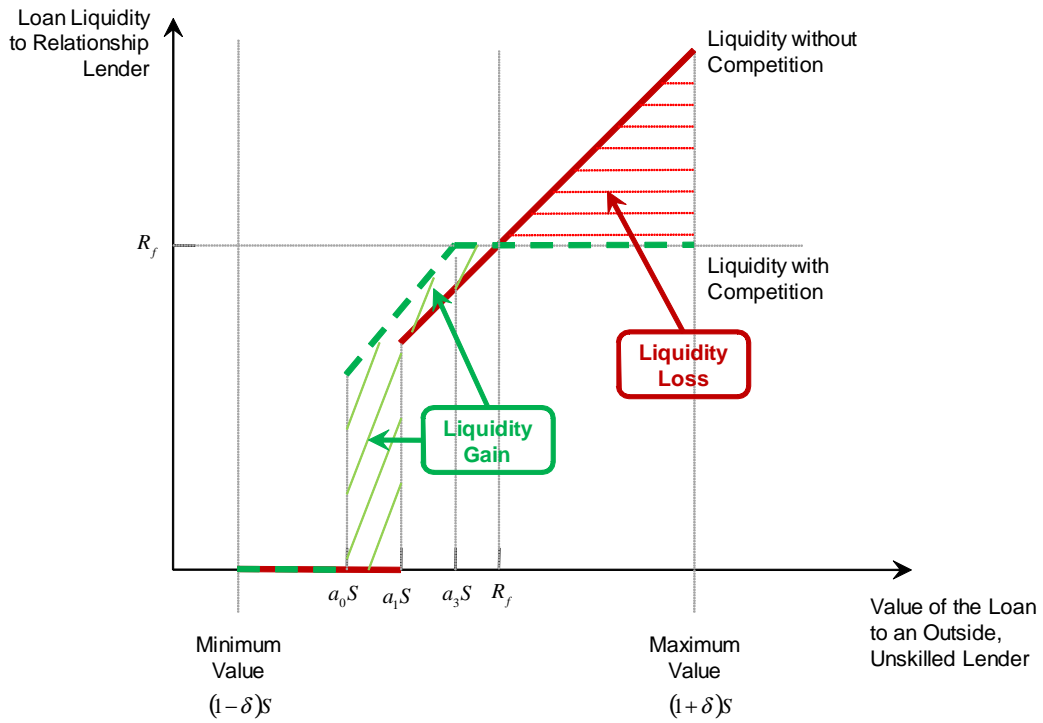
**Proposition 5** *When project payoff and asset values are uncertain:*

- i. Interbank competition is expected to improve the liquidity of an illiquid loan if the expected cost of the liquidation technology for unskilled lenders ( $T$ ) is sufficiently high, i.e.,  $T > \underline{T}_{cc}$  ( $\underline{T}_{cc}$  is defined in the proof). This gain in liquidity increases with  $T$ .*
- ii. The expected gain in the liquidity of the loan decreases with the credit risk of the associated project ( $\delta$ ) if  $\delta > \delta^+$ , and is indeed negative (net loss) if  $\delta > \delta^{++}$  ( $\delta^+$  and  $\delta^{++}$  are defined in the proof).*
- iii. If interbank competition improves (reduces) the expected liquidity of the loan, the magnitude of this improvement (reduction) increases with the degree of competition ( $\Lambda$ ).*

Proposition 5 provides the condition under which interbank competition is expected to improve the liquidity of an illiquid loan, and the possibility of this improvement arises from



a. Credit Risk ( $\delta$ ) is Relatively Low



b. Credit Risk ( $\delta$ ) is Relatively High

Figure 2. The Role of Interbank Competition in the Presence of Credit Risk

the availability of a sufficiently valuable liquidation technology for purchase at a price; a technology that can bring about a greater improvement in an unskilled lender's liquidation skill is considered more valuable and sells at a higher price. Moreover, in the presence of competition, lenders no longer pursue higher liquidity by increasing their exposure to credit risk. This is because interbank competition drives down the lender's profits in the high states, so that the pursuit of credit risk is no longer as profitable. Nevertheless, if the level of credit risk is already high, competition may lead to net loss in liquidity and hence raise liquidity risk. Finally, the magnitude of the impact of interbank competition on loan liquidity, either improvement or reduction, depends on the intensity of competition in the loan market.<sup>24</sup>

### 4.5.2 Condition for Having A Liquid Loan

We recompute the expected cash flow to the impatient lender at date 1 in the presence of competition as follows:

$$E[\tilde{V}_1^{*'}] = \Lambda \left\{ K_0 \cdot \min[C_1 + S', E_1^I] + R_f \cdot \frac{1 + \delta - a_3}{2\delta} \right\} + (1 - \Lambda) \{ A_1 \cdot \min[C_1 + S, E_1^I] \}. \quad (26)$$

Thus, the condition on  $S'$  for this risky and initially illiquid loan to be liquid is

$$\min[C_1 + S', E_1^I] \geq \underline{L}_2 \equiv K_0^{-1} \left\{ R_f \left( \frac{1}{\Lambda} - \frac{1 + \delta - a_3}{2\delta} \right) - \left( \frac{1 - \Lambda}{\Lambda} \right) A_1 \cdot \min[C_1 + S, E_1^I] \right\}. \quad (27)$$

We take the first derivative of  $\underline{L}_2$  with respect to the competition intensity measure  $\Lambda$ :

$$\frac{\partial \underline{L}_2}{\partial \Lambda} = K_0^{-1} \{ A_1 \cdot \min[C_1 + S, E_1^I] - R_f \} \cdot \frac{1}{\Lambda^2}. \quad (28)$$

We have already known that  $A_1 < 1$  (based on the definition of  $A_1$ ) and  $\min[C_1 + S, E_1^I] < R_f$  (Assumption 4). Thus, (28) is negative for all  $\Lambda$ . Since  $\frac{\partial \underline{L}_2}{\partial \Lambda} < 0$ , the condition for a risky loan to be liquid becomes less stringent as the degree of competition increases.

### 4.5.3 The Case of Perfect Competition

We now carefully examine the case in which  $\Lambda = 1$ . This is the case in which the loan market is perfectly competitive and the entrepreneur can always find a competing lender so that Bertrand competition invariably occurs. In order to have  $\Lambda = 1$ , we need  $\lambda = 1$  and/or  $N \rightarrow \infty$ . This perfectly competitive setting shows most clearly how competition affects liquidity and credit risks.

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<sup>24</sup>As in the one-state world, the competitive measure ( $\Lambda$ ) is increasing in both the number of competing banks ( $N$ ) and the probability that each bank is located by the borrowing entrepreneur ( $\lambda$ ). We do not repeat the same results in this section.

**Condition for Having A Liquid Loan:** With  $\Lambda = 1$ , the condition for a risky and initially illiquid loan to be liquid, i.e., Equation (27), becomes

$$\min [C_1 + S', E_1^I] \geq \underline{L}_2 = K_0^{-1} \left[ \frac{a_3 - (1 - \delta)}{2\delta} \right] R_f. \quad (29)$$

We take the first derivative of  $\underline{L}_2$  with respect to the credit risk measure  $\delta$ :

$$\frac{\partial \underline{L}_2}{\partial \delta} = \left( \frac{2}{a_3^2 - a_0^2} \right) R_f > 0. \quad (30)$$

Hence, the lower bound for the loan to be liquid is increasing in  $\delta$ , so it can no longer be true that a loan with higher credit risk is liquid while one with lower credit risk is not.

**Financing Condition:** We discuss in the following lemma the condition for a risky, illiquid loan to be sold in a perfectly competitive loan market (i.e.,  $\Lambda = 1$ ) by the impatient lender at date 1 when  $\tilde{\alpha} \geq a_2$ .

**Lemma 7** *Suppose Assumptions 1-10 hold. Further: (1) cash flows produced by the project and its liquidation values are risky, (2) lending is provided through financial intermediation with complete deposit insurance, and (3) there is perfect interbank competition in the loan market ( $\Lambda = 1$ ). Then, if the following condition holds:*

$$E_1^P \geq \underline{E}_4 \equiv \left\{ \begin{array}{l} R_f \left[ \frac{a_3 - (1 - \delta)}{2\delta} \right] + \frac{\theta}{1 - \theta} \frac{R_l}{R_f} \left[ R_f \left[ \frac{a_3 - (1 - \delta)}{2\delta} \right] - K_0 (C_1 + S') \right] \\ - K_2 (C_1 + S') \end{array} \right\} K_3^{-1} \text{ and}$$

$$C_1 + S' \geq \underline{CS}_4 \equiv \frac{E_1^P R_f}{R_l} + (r_f + \eta) \left( 1 - \frac{R_f}{R_l} \right) \left( \frac{2}{a_3 + a_2} \right), \quad (\text{Condition 4})$$

*the entrepreneur will be financed at date 0, the relationship lender will sell the loan at date 1 when  $\tilde{\alpha} < a_2$ , and the impatient lender will do so even when  $\tilde{\alpha} \geq a_2$ .*

Lemma 7 provides some restrictions on the expected cash payoffs and asset values for the project to be financed when there is perfect competition in the loan market.

Now, we examine how Condition 4 varies with the credit risk measure  $\delta$ . First, we take the first derivative of  $\underline{E}_4$  with respect to  $\delta$ :

$$\frac{\partial \underline{E}_4}{\partial \delta} = \left( \frac{1}{2\delta \cdot K_3} \right) \left( 1 + \frac{\theta}{1 - \theta} \frac{R_l}{R_f} \right) R_f > 0, \quad (31)$$

which means that  $\underline{E}_4$  always increases in  $\delta$ . Meanwhile, it is clear that  $\underline{CS}_4$  does not change with  $\delta$ . Thus, we conclude that a loan with higher credit risk can no longer be made with a lower liquidity premium than a lower-credit-risk loan that has the same expected payoff.

We now summarize these results in the following proposition.

**Proposition 6** *When project payoffs and asset values are uncertain and there is perfect interbank competition in the loan market, a loan that has higher credit risk also has higher liquidity risk. That is, liquidity risk and credit risk are comonotonic in the presence of perfect competition.*

Proposition 6 states that interbank competition decreases the propensity of banks to take additional credit risk in response to liquidity risk. The intuition is as follows. Interbank competition makes the payoffs in the high states (those above  $R_f^2$  at date 2 either with or without the liquidation technology) converge to  $R_f^2$  as unskilled lenders compete to refinance the entrepreneur by offering contracts with lower repayments. This rent-diminishing effect is greater on loans with higher credit risk because they have more high-payoff states than those with lower credit risk. As a result, increasing the credit risk of their loans can no longer provide banks higher expected liquidity. Conversely, if a bank has already taken on excessive credit risk, competition only worsens its liquidity risk.

#### 4.5.4 Discussion of the General Case

Having examined the effect of *perfect* interbank competition on liquidity and credit risks, we now relax the condition  $\Lambda = 1$  and discuss the more general case in which  $\Lambda \in [0, 1]$ . That is, the loan market may be *imperfectly* competitive and the entrepreneur has a probability  $1 - \Lambda$  of not being able to find an alternative lender to fund his project.

First, Proposition 5 characterizes liquidity creation through competition in the general case of  $\Lambda \in [0, 1]$ . It shows that price competition can improve loan liquidity if there exist a sufficiently high price for acquiring higher liquidation skill and if the level of credit risk is *not* too high. This effect is stronger with a higher degree of competition.

Next, in our discussion of the condition for a risky loan to be liquid (Section 4.5.2), we have shown that the lower bound of  $\min [C_1 + S', E_1^I]$  (the maximum liquidity available to the impatient lender at date 1) for the loan to be liquid is decreasing in the degree of competition. That is, a loan is more likely to be liquid when there is more competition.

We then show how *perfect* interbank competition corrects banks' excessive taking of credit risk induced by their liquidity needs (Section 4.5.3). In a setting of *imperfect* competition, the effect stated in Proposition 6 occurs with probability  $\Lambda$ . That is, we conjecture that even with *imperfect* competition, this effect of interbank competition grows stronger as the loan market becomes more competitive.

## 5 Empirical Predictions

Based on our analysis above, we are able to draw the following empirical implications:

1. Loans provided through direct lending (e.g., by the capital market) have lower costs of liquidity to borrowers, compared to those made by banks using deposits, assuming

borrowers are similar in credit risk and project quality (Proposition 1).<sup>25</sup>

2. The liquidity of a loan is positively correlated with the degree of competition and the cost of conducting industry- or firm-specific research (Propositions 3 and 5).
3. A loan tends to be more liquid if the project being financed by the loan produces a higher interim-period cash flow, has a higher liquidation value to the relationship lender, and/or has a lower level of specialization (Proposition 3).
4. When interbank competition is negligible, loans with higher credit risk tend to have better liquidity than loans with lower credit risk, so an increase in liquidity risk induces the bank to increase its credit risk (Proposition 4).
5. In the presence of interbank competition, loans that have better liquidity tend to be those with lower credit risk (Proposition 6 and the conjecture for the general case of imperfect competition). Thus, in banking markets with little interbank competition for loans, there should be a positive correlation between loan liquidity and credit risk, whereas in highly competitive banking markets, there should be a negative correlation between loan liquidity and credit risk.

## 6 Conclusion

The focus of this paper has been on liquidity risk, how it is affected by interbank competition, and how it interacts with credit risk with and without competition.

One of our key results is that interbank competition in the loan market improves the liquidity of loans. This result arises from the existence of a market for the liquidation technology where competing lenders have an opportunity, at a price, to increase their liquidation skill up to the level of the incumbent relationship lender. The result is intuitive – greater interbank competition means a deeper secondary market in which the bank can sell its loans and hence more liquidity.

Competition, however, is not a panacea when it comes to liquidity risk. Some loans may remain illiquid despite introducing interbank competition. There are three possible ways in which this can happen: (1) the projects produce too little cash flows in the interim period to satisfy lenders' liquidity needs, (2) the projects do not provide sufficiently high liquidation values even to the relationship lenders, and (3) the projects are too specialized. In any of these cases, illiquidity can be reduced to some degree through competition, but not enough to make the loans fully liquid.

Another interesting result is that loans with higher credit risk may have lower liquidity risk in the absence of interbank competition. This means that the bank may be induced to

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<sup>25</sup>Costs of illiquidity can be measured empirically by the interest spread charged on the loan, the likelihood of early liquidation, and the probability that a loan application is rejected.

take on excessive credit risk in order to manage its liquidity risk in loan markets that have negligible competition. However, the introduction of interbank competition mitigates the bank's incentive to take on additional credit risk to reduce its liquidity risk because competition makes liquidity risk comonotonic with credit risk. As a result, interbank competition can help reduce the bank's overall risk, which includes credit and liquidity risks. On the other hand, however, if there exists a very high level of credit risk in the loan market, an elevated level of liquidity risk as well as banks' overall risk will accompany with competition.

These results provide potential policy suggestions for regulators. For example, for industries that are highly specialized, it may be useful to contemplate relaxing bank entry restrictions to foster competition and spur liquidity creation. This is also true for highly risky industries in which competition can also reduce banks' credit risks.

For future research, it may be interesting to consider the *dynamic* aspects of the effect of interbank competition on loan liquidity and bank incentives. We can offer some conjectures about how these dynamics might work. An initially illiquid loan becomes liquid as competition motivates unskilled lenders to improve their value-extraction skills. Nevertheless, this improved liquidity is fragile because it generates profit opportunities that attract more competition. When competition lowers profits from the existing loan sufficiently so that the lender's liquidity needs are no longer satisfied (e.g., returns fall below the lender's cost of capital), the lender will invest in a more risky project for higher returns. This more risky loan may not be liquid at the beginning, but competition will create liquidity. The increased liquidity, however, again invites more competition which consequently reduces profits and induces the lender to take even higher credit risk. But credit risk cannot be increased without limit. With competition, the high-end returns from a risky project are quickly lost to competition; even better extraction skills can no longer improve a risky loan's liquidity as indicated in Proposition 5 once its credit risk exceeds a certain level. Eventually the risk-pursuing lenders are trapped by loans that present both high default risk and high liquidity risk. This dynamic story seems to nicely explain the recent subprime crisis.

# A Appendix: Proofs of Lemmas and Propositions

## A.1 Proof of Lemma 1

**Proof.** For an illiquid loan to be made under the condition that the impatient relationship lender will sell the loan at date 1, the entrepreneur's profit maximizing problem is subject to the following constraints:

$$\begin{aligned}
 (IR) \quad & \theta (V_1^I + V_2^I) R_l + (1 - \theta) (V_1^P R_f + V_2^P) \geq \theta R_f R_l + (1 - \theta) R_f^2 \\
 (IC_I) \quad & (V_1^I + V_2^I) R_l \geq V_1^P R_l + V_2^P \\
 (IC_P) \quad & (V_1^I + V_2^I) R_f \leq V_1^P R_f + V_2^P \\
 (\text{Upper bound}) \quad & V_1^I + V_2^I \leq C_1 + S \\
 (\text{Renegotiation-proof}) \quad & V_1^P R_f + V_2^P \leq E_1^P R_f
 \end{aligned}$$

The least expensive way for the entrepreneur to satisfy all these constraints is to set  $V_1^I + V_2^I = C_1 + S$  (due to Assumption 4),  $V_1^P = 0$ , and  $V_2^P = E_1^P R_f$ . Then,  $(IR)$  becomes

$$E_1^P \geq R_f + \frac{\theta}{1 - \theta} \frac{R_l}{R_f} [R_f - (C_1 + S)], \quad (A1)$$

and  $(IC_I)$  becomes

$$C_1 + S \geq \frac{E_1^P R_f}{R_l}, \quad (A2)$$

while  $(IC_P)$  always holds with  $(C_1 + S) R_f \leq E_1^I R_f \leq E_1^P R_f$ .

Combining  $(A1)$  and  $(A2)$  gives us Condition 1.

Now, suppose  $(A2)$  is violated. Then,  $(IC_I)$  does not hold with  $V_2^P = E_1^P R_f$ . To satisfy  $(IC_I)$ , we must have  $V_2^P \leq (C_1 + S) R_l$ . If so, however,  $(IR)$  can never be satisfied while Assumption 5 is true because  $\theta (V_1^I + V_2^I) R_l + (1 - \theta) (V_1^P R_f + V_2^P) \leq (C_1 + S) R_l < \underline{U}$ .

■

## A.2 Proof of Lemma 2

**Proof.** When the project is financed through fully insured deposits, the entrepreneur's profit maximization problem is subject to one more constraint:  $V_1^I, V_1^P \geq r_f + \eta$ . Thus, we can no longer completely back-load payments to the patient lender. That is, we now need to set  $V_1^P = r_f + \eta$  and  $V_1^I = (E_1^P - r_f - \eta) R_f$  instead of  $V_1^P = 0$  and  $V_1^I = E_1^P R_f$  as in the proof of Lemma 1. It is easy to see that conditions derived for the basic model can still satisfy the lender's  $IR$  constraint as well as the  $IC$  constraint for the patient lender when  $V_1^P$  is increased to  $r_f + \eta$ . However, the  $IC$  constraint for the impatient lender becomes  $(C_1 + S) R_l \geq (r_f + \eta) R_l + (E_1^P - r_f - \eta) R_f$ , which then gives us

$$C_1 + S \geq \frac{E_1^P R_f}{R_l} + (r_f + \eta) \left(1 - \frac{R_f}{R_l}\right). \quad (A3)$$

Combining Condition 1 and  $(A3)$  gives us Condition 2. ■

### A.3 Proof of Proposition 1

**Proof.** Proposition 1 follows immediately if we compare Condition 2 to Condition 1. ■

### A.4 Proof of Proposition 2

**Proof.** First, suppose that competition occurs with probability one. The cost for an unskilled lender (if found by the entrepreneur) to refinance is  $R_f$  and the present value of the return is  $\frac{\beta X_2}{R_f}$ . Hence, if  $\beta X_2 \geq R_f^2$ , the unskilled lender will choose to offer a refinancing deal to the entrepreneur. Because there is a price competition between the unskilled lender and either the relationship lender or other unskilled lenders, each of the competing lenders will offer  $Z_2 = R_f^2$  in equilibrium and the entrepreneur will agree to refinance.

No unskilled lenders will attempt to improve their liquidation skill by investing in the liquidation technology because buying the technology costs  $T > 0$  and decreases their return to  $\frac{Z_2}{R_f} - R_f - T$ . Since the equilibrium price is  $Z_2 = R_f^2$ , they would incur losses if they acquired the liquidation technology.

Anticipating this outcome, the relationship lender sets  $Z_2 = R_f^2$  in the initial contract. Since  $Z_2 \leq \beta X_2 < X_2$ , the date-2 repayment is renegotiation-proof and the entrepreneur can commit to paying this amount. Hence, a repayment of  $R_f^2$  is warranted at date 2 to either the relationship lender or an unskilled lender. Then, the date-2 portion of this loan will be sold at  $R_f$ , so that the liquidity of the loan decreases from  $\frac{\beta X_2}{R_f}$  to  $R_f$ .

Meanwhile, when  $S = \frac{\beta X_2}{R_f} \geq R_f$ , the initial loan is liquid because the condition  $L = \min [C_1 + S, E_1^I] \geq R_f$  is satisfied. Even though the liquidity decreases to  $R_f$  in the presence of price competition,  $L \geq R_f$  is still satisfied with  $S = R_f$ . Thus, the loan remains liquid even with a reduced market value.

Now, consider the general case in which such price competition occurs with probability  $\Lambda$ . Then, the relationship lender sets  $Z_2 = \frac{\beta X_2}{R_f}$  in the initial contract but is willing to renegotiate it down to  $R_f^2$  if a competing lender arrives. As a result, the expected liquidity of the loan decreases from  $\frac{\beta X_2}{R_f}$  to  $\Lambda R_f + (1 - \Lambda) \frac{\beta X_2}{R_f}$ . ■

### A.5 Proof of Lemma 3

**Proof.** If  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) X_2 \geq R_f^2$ , then an unskilled, competing lender will first acquire the liquidation technology and then attempt to refinance the entrepreneur's loan because its expected amount of repayment exceeds the refinancing cost only after it acquires the liquidation technology. However, due to the price competition among lenders (as discussed in the proof of Proposition 2), a refinancing deal will not give any lender a repayment above  $R_f^2$  at date 2. Thus, the expected payoff of the unskilled lender who acquires the liquidation technology and successfully refinances the loan is  $R_f - \frac{\beta X_2}{R_f}$  at date 1.

When the liquidation technology is priced at  $T$ , there is a demand of  $n(T)$  homogenous buyers. As a result, each of the buyers has a probability  $\frac{1}{n(T)}$  to win the loan refinancing deal.

Thus, the expected payoff (valued at date 1) from purchasing the liquidation technology is  $\frac{1}{n(T)} \left[ R_f - \frac{\beta X_2}{R_f} \right] - T(\gamma)$ . An unskilled lender will buy the liquidation technology if  $\frac{1}{n(T)} \left[ R_f - \frac{\beta X_2}{R_f} \right] - T(\gamma) \geq 0$ . Thus, the liquidation technology price is set at  $T(\gamma) = \frac{1}{n(T)} \left[ R_f - \frac{\beta X_2}{R_f} \right]$  in equilibrium, which gives the buyer an expected profit of zero.

The total revenue of the monopolist in the liquidation technology market is always  $R_f - \frac{\beta X_2}{R_f}$  ( $= T \cdot n(T)$ ), which does not change with either  $T$  or  $n(T)$ . Offering the liquidation technology to all potential buyers is then a weakly dominant strategy for the monopolist (consider the possibility of new entry if some potential buyers are not served because the liquidation technology price is set too high). Potential buyers of the liquidation technology are the unskilled lenders that have been found by the entrepreneur, and we know the expected number of such competing lenders is  $\lambda N$ . Thus, we obtain  $n(T) = \lambda N$ .

This means that, if  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) X_2 \geq R_f^2$ , the market equilibrium price of the liquidation technology is  $T(\gamma) = \frac{1}{\lambda N} \left[ R_f - \frac{\beta X_2}{R_f} \right]$ . ■

## A.6 Proof of Lemma 4

**Proof.** If  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) X_2 < R_f^2$ , then an unskilled, competing lender will first acquire the liquidation technology and then attempt to buy the loan from an impatient relationship lender. Refinancing the entrepreneur is not a feasible option in this case.

When the technology is priced at  $T$ , there is a demand of  $n(T)$  homogenous buyers. As a result, each of the buyers has a probability  $\frac{1}{n(T)}$  to win the loan purchase bid. Thus, the expected payoff (valued at date 1) from purchasing the technology is  $\frac{\theta}{n(T)} \left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f} - T(\gamma)$ . Here,  $\left[ \bar{\beta}'(\gamma) - \beta \right] X_2$  is the expected increase in the date-2 liquidation value to an unskilled lender who acquires the liquidation technology and successfully buys the loan from the relationship lender. This term must be multiplied by  $\theta$ , the probability of the relationship lender being hit with a liquidity shock, because the loan is not for sale unless the initial lender is impatient. Also, it needs to be discounted by  $R_f$  since the cost of acquiring the liquidation technology is incurred at date 1. An unskilled lender will buy the liquidation technology if  $\frac{\theta}{n(T)} \left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f} - T(\gamma) \geq 0$ . The liquidation technology price is thus set at  $T(\gamma) = \frac{\theta}{n(T)} \left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f}$  in equilibrium, which gives all buyers an expected profit of zero.

As explained in the proof of Lemma 3, the monopolist in the liquidation technology market weakly prefers to offer this technology to all potential buyers. With an expected number of competing lenders found by the entrepreneur being  $\lambda N$ , we obtain  $n(T) = \lambda N$ .

This means that, if  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) X_2 < R_f^2$ , the market equilibrium price of the liquidation technology is  $T(\gamma) = \frac{\theta}{\lambda N} \left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f}$ . ■

## A.7 Proof of Lemma 5

**Proof.** First, suppose  $T(\gamma) > 0$ . Due to the liquidation technology market-clearing condition, we have  $n(T) > 0$ . Then, we must have  $\beta X_2 < R_f^2$ , because otherwise no unskilled lender would invest in the technology. If  $\bar{\beta}'(\gamma) X_2 \geq R_f^2$ , then  $\bar{\beta}'(\gamma) \geq \frac{R_f^2}{X_2} > \beta$ . If  $\bar{\beta}'(\gamma) X_2 < R_f^2$ , then from Lemma 4, we know  $T(\gamma) = \frac{\theta}{\lambda N} [\bar{\beta}'(\gamma) - \beta] \frac{X_2}{R_f}$ . Since  $\lambda N > 0$ ,  $\theta > 0$ ,  $X_2 > 0$ , and  $R_f > 0$ , it must be true that  $\bar{\beta}'(\gamma) > \beta$ . Therefore, in both scenarios, we obtain  $\bar{\beta}'(\gamma) > \beta$ .

Next, suppose  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) > \beta$ . If  $\bar{\beta}'(\gamma) \geq \frac{R_f^2}{X_2} > \beta$ , then  $\bar{\beta}'(\gamma) X_2 \geq R_f^2$ ; from Lemma 3, we get  $T(\gamma) = \frac{1}{\lambda N} \left[ R_f - \frac{\beta X_2}{R_f} \right] > 0$  immediately. Similarly, if  $\frac{R_f^2}{X_2} > \bar{\beta}'(\gamma) > \beta$ , then  $\bar{\beta}'(\gamma) X_2 < R_f^2$ ; from Lemma 4, we get  $T(\gamma) = \frac{\theta}{\lambda N} [\bar{\beta}'(\gamma) - \beta] \frac{X_2}{R_f} > 0$ . ■

## A.8 Proof of Proposition 3

**Proof.** i. From Lemma 5, we know that if  $T(\gamma) > 0$ , then  $\beta X_2 < R_f^2$  and  $\bar{\beta}'(\gamma) > \beta$ . Hence, an unskilled, competing lender will first acquire the liquidation technology and then either refinance the entrepreneur's loan or buy the existing loan from an impatient lender depending on the value of  $\bar{\beta}'(\gamma) X_2$ . Let  $S'$  denote the loan sale price when the entrepreneur finds at least one unskilled lender (i.e., competition actually occurs). Case 2 (Section 3.3.2) shows that if  $\frac{\bar{\beta}'(\gamma) X_2}{R_f} \geq R_f$ , then  $S' = R_f$ ; and Case 3 (Section 3.3.3) shows that if  $\frac{\bar{\beta}'(\gamma) X_2}{R_f} < R_f$ , then  $S' = \frac{\bar{\beta}'(\gamma) X_2}{R_f}$ . Thus, we can write  $S' = \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right]$ . Note that this outcome occurs with probability  $\Lambda$ . This means that the expected loan sale price in the competitive setting is  $S^* \equiv \Lambda S' + (1 - \Lambda) S = S + \Lambda \left\{ \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right] - \frac{\beta X_2}{R_f} \right\}$ . We immediately obtain  $S^* > S$  with  $\Lambda > 0$ ,  $\bar{\beta}'(\gamma) > \beta$ , and  $R_f > \frac{\beta X_2}{R_f}$ . That is, the liquidity of an illiquid loan is expected to increase when there is price competition in the loan market. Then, the lender faces lower liquidity risk (reflected in a higher expected loan sale price) and the entrepreneur bears lower costs of illiquidity (reflected in a lower liquidity premium and a less stringent Condition 2 or Condition 3, depending on whether it is a directly-financed or intermediated loan).

ii. The expected gain in loan liquidity is

$$G \equiv S^* - S = \Lambda \left\{ \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right] - \frac{\beta X_2}{R_f} \right\}, \quad (\text{A4})$$

where  $\Lambda = 1 - (1 - \lambda)^N$ . Taking the first derivative of  $G$  with respect to the competition intensity measures  $N$  and  $\lambda$ , we get

$$\frac{\partial G}{\partial N} = -\ln(1 - \lambda) \cdot (1 - \lambda)^N \cdot \left\{ \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right] - \frac{\beta X_2}{R_f} \right\} > 0, \quad (\text{A5})$$

$$\frac{\partial G}{\partial \lambda} = N(1 - \lambda)^{N-1} \cdot \left\{ \min \left[ \frac{\bar{\beta}'(\gamma) X_2}{R_f}, R_f \right] - \frac{\beta X_2}{R_f} \right\} > 0. \quad (\text{A6})$$

Thus, the liquidity created by competition increases in both  $N$  and  $\lambda$ .

iii. Without interbank competition, a loan is liquid if  $L = \min [C_1 + S, E_1^I] \geq R_f$ . Now, in the presence of competition, this condition becomes  $L^* = \min \left\{ C_1 + S^*, \max \left[ X_1, S^*, \frac{X_2}{R_l} \right] \right\} \geq R_f$ . With Assumption 7, we can further simplify the condition to be  $L^* = C_1 + S^* \geq R_f$ . We discuss below different cases regarding  $S^*$  and show that the conditions in Proposition 3 satisfy  $L^* \geq R_f$  and hence make an initially illiquid loan liquid.

(a) If  $\frac{\bar{\beta}'(\gamma)X_2}{R_f} \geq R_f$ , then  $S' = R_f$ . From Lemma 3, we know that  $T(\gamma) = T_1$ . In order to satisfy  $L^* \geq R_f$ , we need to have

$$C_1 + S^* = C_1 + \Lambda R_f + (1 - \Lambda) \frac{\beta X_2}{R_f} \geq R_f, \quad (\text{A7})$$

which gives us the first condition

$$\Lambda \geq 1 - \frac{C_1}{R_f - \beta X_2 / R_f}. \quad (\text{A8})$$

(b) If  $\frac{\bar{\beta}'(\gamma)X_2}{R_f} < R_f$ , then  $S' = \frac{\bar{\beta}'(\gamma)X_2}{R_f}$ . From Lemma 4, we know that  $T(\gamma) = T_2 < T_1$ . In order to satisfy  $L^* \geq R_f$ , we need to have

$$C_1 + S^* = C_1 + \Lambda \cdot \frac{\bar{\beta}'(\gamma)X_2}{R_f} + (1 - \Lambda) \frac{\beta X_2}{R_f} \geq R_f. \quad (\text{A9})$$

Lemma 4 gives us  $T(\gamma) = \frac{\theta}{\lambda N} \left[ \bar{\beta}'(\gamma) - \beta \right] \frac{X_2}{R_f}$  when  $\frac{\bar{\beta}'(\gamma)X_2}{R_f} < R_f$ . Hence, we can express the improvement in the liquidation skill of an unskilled lender as follows:

$$\bar{\beta}'(\gamma) - \beta = T(\gamma) \left( \frac{\lambda N \cdot R_f}{\theta X_2} \right). \quad (\text{A10})$$

Substituting  $\bar{\beta}'(\gamma) - \beta$  in (A9) with (A10), we obtain the second condition

$$T(\gamma) \geq \frac{\theta [R_f - C_1 - \beta X_2 / R_f]}{\Lambda \cdot \lambda N}. \quad (\text{A11})$$

■

## A.9 Proof of Lemma 6

**Proof.** When  $\tilde{\alpha} < a_1$ , the payoff to the lender is always zero, whether it is patient or impatient. When  $a_1 \leq \tilde{\alpha} < a_2$ , the lenders of both types sell the loan and obtain  $\tilde{\alpha}(C_1 + S)$  at date 1. When  $\tilde{\alpha} \geq a_2$ , the impatient lender sells the loan while the patient lender keeps the loan. Applying the same reasoning here as in the proofs of Lemmas 1 and 2, we set  $V_1^I + V_2^I = \tilde{\alpha}(C_1 + S)$ ,  $V_1^P = r_f + \eta$  and  $V_1^P = (\tilde{\alpha}E_1^P - r_f - \eta) R_f$  when  $\tilde{\alpha} \geq a_2$ .

Then, the (IR) constraint can be written as

$$\theta [A_1 (C_1 + S) R_l] + (1 - \theta) [K_1 (C_1 + S) R_f + A_2 E_1^P R_f] \geq \theta R_f R_l + (1 - \theta) R_f^2, \quad (\text{A12})$$

which gives us the condition

$$E_1^P \geq \left\{ R_f + \frac{\theta}{1-\theta} \frac{R_l}{R_f} [R_f - A_1 (C_1 + S)] - K_1 (C_1 + S) \right\} A_2^{-1}. \quad (A13)$$

Next, the  $(IC_I)$  constraint can be written as

$$(A_1 - K_1) (C_1 + S) R_l \geq A_2 E_1^P R_f + (r_f + \eta) (R_l - R_f) \left( \frac{1 + \delta - a_2}{2\delta} \right), \quad (A14)$$

which gives us the condition

$$C_1 + S \geq \frac{E_1^P R_f}{R_l} + (r_f + \eta) \left( 1 - \frac{R_f}{R_l} \right) \left( \frac{2}{1 + \delta + a_2} \right). \quad (A15)$$

Lastly, the  $(IC_P)$  constraint is

$$(A_1 - K_1) (C_1 + S) R_f \leq A_2 E_1^P R_f, \quad (A16)$$

which always holds with  $K_1 = A_1 - A_2$  and  $C_1 + S \leq E_1^I \leq E_1^P$ .

Combining (A13) and (A15) gives us Condition 3. ■

## A.10 Proof of Proposition 4

**Proof.** i. Based on the comparison between Condition 2 (for riskless projects) and Condition 3 (for risky projects) in Section 4.4, we know that (1) if  $C_1 + S > R_f (1 - A_2) K_1^{-1}$ , then  $\underline{E}_2 > \underline{E}_3$  and (2) if  $\delta > \delta_1 = 1 - a_2$  (which always holds in our setting with  $\delta > 1 - a_1$  and  $a_1 < a_2$ ), then  $\underline{CS}_2 > \underline{CS}_3$ . Thus, the condition for an illiquid, risky loan to be made may be less stringent than that for an illiquid but riskless loan whose certain payoff equals the expected value of the payoff on the risky loan. As a result, the risky project may be able to obtain financing while the riskless project does not, and the liquidity premium of this risky loan may be lower than that of the riskless loan.

ii. Based on the comparative statics of the liquidity conditions for risky projects in Section 4.4, we know that (1) if  $\delta > \delta_2 = \sqrt{1 - a_1^2}$ ,  $\underline{L}_1$  decreases in  $\delta$ , (2) if  $\delta > \delta_3 = \sqrt{\frac{1-\theta}{\theta} \frac{R_f}{R_l} (a_2^2 - a_1^2) + (1 - a_1^2)}$ ,  $\underline{E}_3$  decreases in  $\delta$ , and (3)  $\underline{CS}_3$  decreases in  $\delta$  when  $E_1^P$  is held constant. Thus, among two risky loans with equal expected payoffs, the condition for the riskier loan to be liquid or to be made may be less stringent than that for the less risky loan. As a result, the riskier project may obtain financing when the less risky project does not, and the liquidity premium of the riskier loan may be lower than that of the less risky loan. ■

## A.11 Proof of Proposition 5

**Proof.** i. Using  $T = T_2 = \frac{\theta}{\lambda N} \left[ \bar{\beta}' - \beta \right] \frac{X_2}{R_f}$ , we can write

$$S' = \frac{\lambda N}{\theta} \cdot T + S, \quad (A17)$$

and

$$E(\tilde{G}) = \Lambda \left[ K_0 \cdot \frac{\lambda N}{\theta} \cdot T + \left( \frac{1 + \delta - a_3}{2\delta} \right) R_f - (A_1 - K_0) \cdot S \right]. \quad (A18)$$

For competition to create liquidity, we need  $E(\tilde{G}) > 0$ , which can be satisfied if the following is true:

$$G_{\Lambda=1} \equiv K_0 \cdot \frac{\lambda N}{\theta} \cdot T + \left( \frac{1 + \delta - a_3}{2\delta} \right) R_f - (A_1 - K_0) \cdot S > 0. \quad (A19)$$

The condition (A19) holds if

$$T > \underline{T}_{cc} \equiv \frac{\theta}{K_0 \cdot \lambda N} \left[ (A_1 - K_0) \cdot S - \left( \frac{1 + \delta - a_3}{2\delta} \right) R_f \right]. \quad (A20)$$

We take the first derivative of  $E(\tilde{G})$  with respect to  $T$ :

$$\frac{\partial E(\tilde{G})}{\partial T} = \Lambda \left( K_0 \cdot \frac{\lambda N}{\theta} \right) > 0. \quad (A21)$$

Since  $\frac{\partial E(\tilde{G})}{\partial T} > 0$ , the expected gain in the liquidity of the loan is increasing in  $T$ .

ii. We take the first derivative of  $E(\tilde{G})$  with respect to  $\delta$ :

$$\frac{\partial E(\tilde{G})}{\partial \delta} = \frac{\Lambda}{4\delta^2} \left[ - (a_3^2 - a_0^2) S' + 2(a_3 - 1) R_f - (\delta^2 - 1 + a_1^2) S \right]. \quad (A22)$$

We have  $\frac{\partial E(\tilde{G})}{\partial \delta} < 0$  if  $\delta > \delta^+ \equiv \sqrt{1 - a_1^2 + \frac{2(a_3-1)R_f - (a_3^2 - a_0^2)S'}{S}}$ . Hence, the expected liquidity gain is decreasing in  $\delta$  if  $\delta > \delta^+$ .

For  $E(\tilde{G}) < 0$ , we solve the following inequality:

$$\Lambda \left[ K_0 \cdot S' + \left( \frac{1 + \delta - a_3}{2\delta} \right) R_f - A_1 \cdot S \right] < 0, \quad (A23)$$

which is equivalent to

$$S \cdot \delta^2 - 2(R_f - S)\delta + (1 - a_1^2)S + 2(a_3 - 1)R_f - (a_3^2 - a_0^2)S' > 0. \quad (A24)$$

The solution to (A24) is

$$\delta > \delta^{++} \equiv \left[ R_f - S + \sqrt{R_f^2 + a_1^2 S - 2a_3 R_f S + (a_3^2 - a_0^2) S S'} \right] S^{-1} \quad (A25)$$

iii. We take the first derivative of  $E(\tilde{G})$  with respect to  $\Lambda$  and get  $\frac{\partial E(\tilde{G})}{\partial \Lambda} = G_{\Lambda=1}$ . Hence, if  $G_{\Lambda=1} > 0$  ( $< 0$ ), liquidity is created (reduced) by competition. This improvement (reduction) is increasing in  $\Lambda$ . ■

## A.12 Proof of Lemma 7

**Proof.** All scenarios of  $\tilde{\alpha}$  stay the same as in the proof of Lemma 5 except that (1) when  $\tilde{\alpha} \geq a_3$ , the lenders of both types will receive  $R_f$  at date 1 as unskilled lenders will offer to refinance the entrepreneur's loan either with or without acquiring the liquidation technology, and (2) when  $a_0 \leq \tilde{\alpha} < a_3$ , the maximum loan sale price is now  $\tilde{S}'$  in the presence of competition (as opposed to  $\tilde{S}$  when there is no competition) as unskilled lenders will acquire the liquidation technology and then compete to buy the loan from the relationship lender. Applying the same reasoning as before, we set  $V_1^I + V_2^I = \tilde{\alpha} (C_1 + S')$ ,  $V_1^P = r_f + \eta$  and  $V_2^P = (\tilde{\alpha} E_1^P - r_f - \eta) R_f$  when  $a_2 \leq \tilde{\alpha} < a_3$ .

The (IR) constraint now can be written as

$$\begin{aligned} & \theta \left[ K_0 (C_1 + S') R_l + \left( \frac{1 + \delta - a_3}{2\delta} \right) R_f R_l \right] \\ & + (1 - \theta) \left[ K_2 (C_1 + S') R_f + K_3 E_1^P R_f + \left( \frac{1 + \delta - a_3}{2\delta} \right) R_f^2 \right] \\ & \geq \theta R_f R_l + (1 - \theta) R_f^2, \end{aligned} \quad (A26)$$

which then gives us the condition

$$E_1^P \geq \left\{ R_f \left[ \frac{a_3 - (1 - \delta)}{2\delta} \right] + \frac{\theta}{1 - \theta} \frac{R_l}{R_f} \left[ R_f \left[ \frac{a_3 - (1 - \delta)}{2\delta} \right] - K_0 (C_1 + S') \right] - K_2 (C_1 + S') \right\} K_3^{-1}. \quad (A27)$$

Then, we examine the ( $IC_I$ ) constraint:

$$(K_0 - K_2) (C_1 + S') R_l \geq K_3 E_1^P R_f + (r_f + \eta) (R_l - R_f) \frac{a_3 - a_2}{2\delta}. \quad (A28)$$

Since  $K_3 = K_0 - K_2$ , (A28) gives us the condition

$$C_1 + S' \geq \frac{E_1^P R_f}{R_l} + (r_f + \eta) \left( 1 - \frac{R_f}{R_l} \right) \left( \frac{2}{a_3 + a_2} \right). \quad (A29)$$

Lastly, the ( $IC_P$ ) constraint is

$$(K_0 - K_2) (C_1 + S') R_f \leq K_3 E_1^P R_f, \quad (A30)$$

which always holds with  $K_3 = K_0 - K_2$  and  $C_1 + S' \leq E_1^I \leq E_1^P$  (due to Assumption 7).

Combining (A27) and (A29) gives us Condition 4. ■

## A.13 Proof of Proposition 6

**Proof.** First, the lower bound of the liquidity condition  $\underline{L}_2$  always increases in  $\delta$  with  $\frac{\partial \underline{L}_2}{\partial \delta} > 0$ , and hence, there can no longer be the case in which a loan with higher credit risk is liquid while a loan with lower credit risk is not. Then, Condition 4 only gets tighter with higher  $\delta$  since  $\frac{\partial E_1^P}{\partial \delta} > 0$  for all  $\delta$ 's and  $\underline{CS}_4$  does not change with  $\delta$ . This implies that, in the presence of perfect interbank competition, loans with higher credit risk (higher  $\delta$ ) also have higher liquidity risk. ■

## References

- [1] Acharya, Viral V., Denis Gromb and Tanju Yorulmazer, 2007a, Failure in the market for liquidity transfers and the origins of central banking, Working paper.
- [2] Acharya, Viral V., Denis Gromb and Tanju Yorulmazer, 2007b, Imperfect competition in the inter-bank market for liquidity, Working paper.
- [3] Acharya, Viral V. and Lasse Heje Pedersen, 2005, Asset pricing with liquidity risk, *Journal of Financial Economics*, Vol. 77, No. 2, 375-410.
- [4] Acharya, Viral V. and Stephen Schaefer, 2006, Liquidity risk and correlation risk: Implications for risk management, Working paper.
- [5] Acharya, Viral V. and S. Viswanathan, 2007, Moral hazard, collateral and liquidity, *FEN Banking & Financial Institutions Working Paper Series*, Vol. 15, No. 9.
- [6] Allen, Franklin and Douglas Gale, 2000, *Comparing financial systems*, MIT Press.
- [7] Allen, Franklin and Douglas Gale, 2004, Competition and stability, *Journal of Money, Credit and Banking*, Vol. 36, No. 3, 453-480.
- [8] Besanko, David and Anjan V. Thakor, 1993, Relationship banking, deposit insurance and bank portfolio choice, *Capital Markets and Financial Intermediation* (Colin Mayer and Xavier Vives, eds.), Cambridge University Press, 292-319.
- [9] Bhattacharya, Sudipto and Anjan V. Thakor, 1993, Contemporary banking theory, *Journal of Financial Intermediation*, Vol. 3, No. 1, 2-50.
- [10] Boot, Arnoud W.A. and Stuart I. Greenbaum, 1993, Bank regulation, reputation and rents: Theory and policy implications, *Capital Markets and Financial Intermediation* (Colin Mayer and Xavier Vives, eds.), Cambridge University Press, 262-285.
- [11] Boot, Arnoud W.A. and Matej Marinč, 2007, Competition and entry in banking: Implications for capital regulation, Working paper.
- [12] Boyd, John H. and Gianni De Nicoló, 2005, The theory of bank risk taking and competition revisited, *Journal of Finance*, Vol. 60, No. 3, 1329-1343.
- [13] Boyd, John H., Gianni De Nicoló and Abu M. Jalal, 2007, Bank competition, risk, and asset allocations: New theory and new evidence, Working paper.
- [14] Calomiris, Charles W. and Charles M. Kahn, 1991, The role of demandable debt in structuring optimal banking arrangements, *American Economic Review*, Vol. 81, No. 3, 497-513.
- [15] Caminal, Ramón and Carmen Matutes, 1997, Can competition in the credit market be excessive?, *CEPR Discussion Papers*, No. 1725.

- [16] Caminal, Ramón and Carmen Matutes, 2002, Market power and banking failures, *International Journal of Industrial Organization*, Vol. 20, No. 9, 1341-1361.
- [17] Carletti, Elena, Competition and regulation in banking, *Financial Intermediation, Corporate Finance, and the Capital Market* (Arnoud W.A. Boot and Anjan V. Thakor, eds.), Elsevier, Forthcoming (2008).
- [18] Chari, V. V. and Ravi Jagannathan, 1988, Banking panics, information, and rational expectations equilibrium, *Journal of Finance*, Vol. 43, No. 3, 749-761.
- [19] Cordella, Tito and Eduardo Levy Yeyati, 2002, Financial opening, deposit insurance, and risk in a model of banking competition, *European Economic Review*, Vol. 46, No. 3, 471-485.
- [20] Degryse, Hans and Steven Ongena, Competition and regulation in the banking sector: A review of the empirical evidence on the sources of bank rents, *Financial Intermediation, Corporate Finance, and the Capital Market* (Arnoud W.A. Boot and Anjan V. Thakor, eds.), Elsevier, Forthcoming (2008).
- [21] Diamond, Douglas W. and Philip H. Dybvig, 1983, Bank runs, deposit insurance, and liquidity, *Journal of Political Economy*, Vol. 91, No. 3, 401-419.
- [22] Diamond, Douglas W. and Raghuram G. Rajan, 2001, Liquidity risk, liquidity creation, and financial fragility: A theory of banking, *Journal of Political Economy*, Vol. 109, No. 2, 287-327.
- [23] Edwards, Franklin R. and Frederic S. Mishkin, 1995, The decline of traditional banking: Implications for financial stability and regulatory policy, *Federal Reserve Bank of New York Economic Policy Review*, July, 27-45.
- [24] Gale, Douglas and Xavier Vives, 2002, Dollarization, bailouts and the stability of the banking system, *Quarterly Journal of Economics*, Vol. 117, No. 2, 467-502.
- [25] Goldstein, Itay and Ady Pauzner, 2005, Demand-deposit contracts and the probability of bank runs, *Journal of Finance*, Vol. 60, No. 3, 1293-1327.
- [26] Gorton, Gary, 1985, Bank suspension of convertibility, *Journal of Monetary Economics*, Vol. 15, No. 2, 177-193.
- [27] Gorton, Gary, 1988, Banking panics and business cycles, *Oxford Economic Papers*, Vol. 40, No. 4, 751-781.
- [28] Gorton, Gary and Andrew Winton, 2000, Liquidity provision, bank capital, and the macroeconomy, Working paper.
- [29] Holmström, Bengt and Jean Tirole, 1998, Private and Public Supply of Liquidity, *Journal of Political Economy*, Vol. 106, No. 1, 1-40.

- [30] Jacklin, Charles J. and Sudipto Bhattacharya, 1988, Distinguishing panics and information-based bank runs: Welfare and policy implications, *Journal of Political Economy*, Vol. 96, No. 3, 568-592.
- [31] Kashyap, Anil K., Raghuram Rajan and Jeremy C. Stein, 2002, Banks as liquidity providers: An explanation for the coexistence of lending and deposit-taking, *Journal of Finance*, Vol. 57, No. 1, 33-73.
- [32] Keeley, Michael C., 1990, Deposit insurance, risk and market power in banking, *American Economic Review*, Vol. 80, No. 5, 1183-1200.
- [33] Matutes, Carmen and Xavier Vives, 1996, Competition for deposits, fragility, and insurance, *Journal of Financial Intermediation*, Vol. 5, No. 2, 184-216.
- [34] Matutes, Carmen and Xavier Vives, 2000, Imperfect competition, risk taking and regulation in banking, *European Economic Review*, Vol. 44, No. 1, 1-34.
- [35] Myerson, Roger B., 1979, Incentive compatibility and the bargaining problem, *Econometrica*, Vol. 47, No. 1, 61-73.
- [36] Petersen, Mitchell A. and Raghuram G. Rajan, 1995, The effect of credit market competition on lending relationships, *Quarterly Journal of Economics*, Vol. 110, No. 2, 407-443.
- [37] Postlewaite, Andrew and Xavier Vives, 1987, Bank runs as an equilibrium phenomenon, *Journal of Political Economy*, Vol. 95, No. 3, 485-491.
- [38] Rochet, Jean-Charles and Xavier Vives, 2004, Coordination failures and the lender of last resort: Was Bagehot right after all?, *Journal of the European Economic Association*, Vol. 2, No. 6, 1116-1147.
- [39] Vives, Xavier, Competition and regulation in banking, *Financial Intermediation, Corporate Finance, and the Capital Market* (Arnoud W.A. Boot and Anjan V. Thakor, eds.), Elsevier, Forthcoming (2008).
- [40] Wagner, Wolf, The homogenization of the financial system and financial crises, *Journal of Financial Intermediation*, Forthcoming (2008).