Rational Expectations in Urban Economics*

Marcus Berliant† and Chia-Ming Yu‡

October 1, 2008

Abstract: Canonical analysis of the classical general equilibrium model demonstrates the existence of an open and dense subset of standard economies that possess fully-revealing rational expectations equilibria. This paper shows that the analogous result is not true in urban economies. An open subset of economies where none of the rational expectations equilibria fully reveal private information is found. There are two important pieces. First, there can be information about a location known by a consumer who does not live in that location in equilibrium, and thus the equilibrium rent does not reflect this information. Second, if a consumer's utility depends only on information about their resided location, perturbations of utility naturally do not incooperate information about other locations. Existence of a non-fully revealing rational expectations equilibrium is proved. Location can prevent housing prices from transmitting information from informed to uninformed households, resulting in an inefficient outcome. (*JEL Classifications*: D51; D82; R13)

Keywords: Urban Economics; General Equilibrium; Private Information; Rational Expectations.

---

*We would like to thank John Conlon for his valuable comments. We acknowledge financial support from Center for Research in Economics and Strategy (CRES), in the Olin Business School, Washington University in St. Louis.

†Department of Economics, Washington University, Campus Box 1208, 1 Brookings Drive, St. Louis, MO 63130-4899 Phone: (1-314) 935-8486, Fax: (1-314) 935-4156, e-mail: berliant@artsci.wustl.edu

‡Department of Economics, Washington University, Campus Box 1208, 1 Brookings Drive, St. Louis, MO 63130-4899 Phone: (1-314) 935-5670, Fax: (1-314) 935-4156, e-mail: cyua@artsci.wustl.edu

1 Introduction

1.1 Motivation

People can never fully comprehend the quality and the circumstances of a city until they experience a significant part of their life living in that city. Information on physical amenities of a city (i.e., weather, parks, museums, crime, traffic jams) is easily acquired by both consumers and researchers, so there is institutional and academic work on the quality of life in cities.[1] However, people cannot completely ensure that they choose the right city or location within the city for their family before they start experiencing life there. For example, there could be uncertainty about the quality of schools, congestion of commuting routes contingent on resident and business location, or even major highway closures. Current occupants of the city, or people with friends living in the city, might have information that others don't have. Moreover, even though the current environment of the city can be understood, it is not surprising that the future developments of cities are not known with certainty, but might be known better by current occupants.[2]

On the one hand, information about life in a city is reflected in the demand for and thus the price of housing in the city.[3] Since people are rational in understanding and using the relationship associating a specific state of nature with a specific equilibrium price, depending on what model people have in mind for how equilibrium prices are determined, the price of housing can be a signal for people in choosing a city best suited to their life style. Recall that the concept of rational expectations equilibrium requires agents

[1]For example, Rosen (1979), Roback (1982), and Blomquist, Berger, and Hoehn (1988) develop the quality of life index for urban areas (QOLI), that measures or implicitly prices the value of local amenities in urban areas.

[2]For example, Cronon (1991) discusses the success of Chicago in surpassing other competitive cities, such as St. Louis, in the early development of the Midwest.

[3]It can also be reflected in wages, but for simplicity we focus on rent.

to use models that are not obviously controverted by their observations of the market. Therefore, the question of whether the price of housing can play a significant role in transmitting information from informed people to uninformed people not only addresses the question of the efficiency of housing markets, but is also related to the issue of the existence of rational expectations equilibrium in urban economics.

Available information is utilized by agents in a rational expectations equilibrium, especially the information conveyed by equilibrium prices. Radner (1979) shows that in a particular asset trading model, if the number of states of initial information is finite then, generically, rational expectations equilibria exist where all traders' private initial information is revealed. In contrast to Radner's model, that fixes state-dependent preferences and then focuses on the information concerning traders' conditional probabilities of various events, Allen (1981) considers a space of economies that is defined by state-dependent preferences and confirms Radner's conclusion in that context. When state space is infinite, Allen (1981) shows that the generic existence of fully-revealing rational expectations equilibria depends on the condition that the price space must have at least as high a dimension as the state space. Jordan (1980) considers a model where information revealed by endogenous variables can be affected by expectations, and then characterizes the data that allow the generic existence of rational expectations equilibria. Jordan concludes that unless the public prediction is based on a very narrow class of data, a statistically correct expectation may fail to exist even for otherwise well-behaved economies.

The existence of rational expectations equilibria where prices do not fully reveal the state of nature motivates the development of this paper. As shown in standard general equilibrium models in literature, fully revealing rational expectations equilibrium demonstrates the efficiency of market prices in information transmission. The cases where the rational expectations equilibrium is not fully revealing are more interesting, for they admit a positive value of private information (that cannot be learned by observing prices) and space for discussing purchases of and strategic behaviors using private information. In opposition to standard models, this paper focuses on the generic existence of non-fully revealing rational expectations equilibrium. In contrast with Allen (1981), who proves the existence of an open and dense subset of economies that possess fully-revealing rational expectations equilibria in the standard general equilibrium model, this paper shows that the analogous result does not hold in urban economies. An open subset of economies is found, where all the rational expectations equilibria of these economies do not fully reveal private information.

Though in different settings, the common intuition behind these economies is consistent. First of all, households' bid rents reflect their ex ante valuations for housing, and the expected valuations reflect households' information (and their prior distributions) about the states. However, the equilibrium bid rent reveals only the winner's valuation, instead of being determined by all households' valuations. Therefore, in urban economics, *the equilibrium price of land reflects only the ex ante valuation and the information of the household with the highest willingness-to-pay for a location.* Conversely, the standard general equilibrium model has aggregate excess demand that is dependent on every household's demand. This generates complete information revelation in equilibrium generically, if there are enough prices. The difference between the models is due to the standard assumption in urban economics that each person can be in only one place at one time. In this circumstance, the equilibrium price might not fully reveal households' private information, even if there are many prices and few states. For example, if in equilibrium a household living in one location has information about another location, this information might not be verified in equilibrium rents.

The other important component, that yields an open set of economies with not all information revealed in equilibrium, concerns perturbations of utility functions. The set of states affecting utility of households living in one location is assumed to be different from the set of in another location; in other words, we use a product structure in state space. This is what we mean when we say information is local. Thus, when we consider perturbations of utility functions, we do not allow the utility of households living in one location to depend even a little on states belonging to other locations. This is what we mean when we say perturbations are spatially local.

The model that we present covers both within-city locations and the comparison of different cities, though the latter case is the focus of this paper. This paper is organized as follows: Two explicit examples give the intuition behind the non-existence of fully-revealing rational expectations equilibrium in Section 2. For generic results, in Section 3, we find an open subset of economies with no fully-revealing rational expectations equilibrium, provided that perturbations are spatially local. In Section 4, the existence of rational expectations equilibrium is demonstrated. When all information is local, there exists a unique non-fully revealing rational expectations equilibrium. When all information is not local, there exists a fully-revealing rational expectations equilibrium. When spatially non-local perturbations are considered, the results are the same as the ones in standard general equilibrium models, namely generic existence of fully revealing rational expectations equilibrium. In this case, generically information is not local.

2 The Examples

Before stating formally and proving the results, let us examine a few examples. In the first example, one of the households has full information, whereas the other has no information. In the second example, both households have partial information about the states of nature in different locations. In both examples, the equilibrium prices are the same in different states, and hence illustrate an economy where the rational expectations equilibria do not fully reveal the private information of households. Examples similar to these appear in the literature on rational expectations in the standard general equilibrium model, though in that literature they belong to the complement of a generic set, and have a very different flavor.

2.1 The Framework

Suppose there are two households indexed by $j = 1, 2$ and two cities, named x and y, with fixed land supply of $\bar{x}$ and $\bar{y}$, respectively. We consider the case where consumers obtain different utility from living in x or living in y. These could either be areas within a city or two different cities. Beside locations, each household has to choose the lot size of his/her house in city k, denoted by s_{jk}, and the consumption of composite good z_{jk}, $k = x, y$, $j = 1, 2$. Since it is impossible to consume a house at the same instant in two locations, $s_{jx} > 0$ implies $s_{jy} = 0$, and $s_{jy} > 0$ implies $s_{jx} = 0$. To placate urban economists, we shall introduce a commuting cost, but all of our arguments hold when commuting cost is set to zero and there is only a utility difference between locations. Consider city x as a core-city and y as a periphery-city. Following Fujita, Krugman, and Venables (2001), there is only commuting from y to x. Denote the unit commuting cost to be t and suppose that the distance between two cities is 1. Then household j has a consumption of composite good z_{jk} in city k, and bears a commuting cost $T_x = 0$ in x or $T_y = t$ in y.

There are two states in each city, $\omega_k \in \Omega_k \equiv \{H, L\}$, $k = x, y$, representing preference differences in our model, each realized with a probability 1/2. Furthermore, the states in the two cities are not correlated. What each agent observes are events that are subsets of $\Omega \equiv \Omega_x \times \Omega_y$. Denote $\omega \equiv \omega_x \times \omega_y$ to be the element in Ω. Suppose that household 1 has no information, and household 2 knows what the state will be. That is, households' information are represented by $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y\}$, $\mathcal{F}_2 = \{\phi, \{H\}, \{L\}, \Omega_x\} \times \{\phi, \{H\}, \{L\}, \Omega_y\}$ which are sub-σ-fields of $\mathcal{F}$, where $\mathcal{F} \equiv \mathcal{F}_1 \vee \mathcal{F}_2$ is the smallest σ-field generated by the class $\mathcal{F}_1 \bigcup \mathcal{F}_2$ of subsets of $\Omega = \{HH, HL, LH, LL\}$.[4] Everything except the true state is common knowledge, so households are assumed to know the relationship between states and prices.

Suppose also that household 1's utility is state-dependent but the utility function of household 2 is independent of states. To focus on an exchange economy, standard in both rational expectations general equilibrium and urban economics models, suppose that every household earns a fixed income Y of composite good. Let p_k denote the price per unit of land in city k, $k = x, y$, and normalize the price of freely mobile composite consumption good to be 1. Households can augment their private information by and only by using the information conveyed by prices.[5] The rents are collected and consumed by one landlord who owns all the land and whose utility is $u_L(s_{Lx}, s_{Ly}, z_L) = z_L$ in all states. The landlord is endowed with an inelastic supply of housing in both cities.

Each household can consume housing in only one city. In state ω, given $(s_{jx}, s_{jy}, z_{jx}, z_{jy})$, the ex post utility function of household j is

$$u_j^\omega(s_{jx}, s_{jy}, z_{jx}, z_{jy}) = \max\{\alpha_j^\omega \ln(s_{jx}) + \ln(z_{jx}),\ \beta_j^\omega \ln(s_{jy}) + \ln(z_{jy})\},$$

[4]Following Aumann (1976), the join $\mathcal{F}_1 \vee \mathcal{F}_2$ denotes the coarsest common refinement of $\mathcal{F}_1$ and $\mathcal{F}_2$.

[5]When households condition their expectations on additional market variables, the equilibrium concept is defined as a generalized rational expectations equilibrium; see Allen (1998).

$\omega \in \Omega$. Given information structure $\mathcal{F}_1$, the superscripts of household 1's allocation can be ignored for simplicity. The optimization problem for household 1 is to maximize expected utility subject to the budget constraint:[6]

$$\begin{aligned}
\max_{s_{1x},s_{1y},z_{1x},z_{1y}} \quad & Eu_1(s_{1x}, s_{1y}, z_{1x}, z_{1y}|\mathcal{F}_1) \\
& = \max\{E[\alpha_1^\omega \ln(s_{1x}) + \ln(z_{1x})|\mathcal{F}_1],\ E[\beta_1^\omega \ln(s_{1y}) + \ln(z_{1y})|\mathcal{F}_1]\} \\
\text{s.t.} \quad & p_x s_{1x} + p_y s_{1y} + z_{1x} + z_{1y} + \ulcorner \frac{s_{1y}}{s_{1x}+s_{1y}} \urcorner t \leq Y, \\
& s_{1k}\, s_{1l} = 0,\ s_{1k}\, z_{1l} = 0,\ z_{1k}\, z_{1l} = 0, \\
& s_{1k}, z_{1k} \geq 0, \quad \forall k, l = x, y,\ k \neq l;
\end{aligned}$$

In contrast, since household 2's utility is state-independent, his/her optimization problem is for all $\omega \in \Omega$

$$\begin{aligned}
\max_{s_{2x}^\omega,s_{2y}^\omega,z_{2x}^\omega,z_{2y}^\omega} \quad & u_2^\omega(s_{2x}^\omega, s_{2y}^\omega, z_{2x}^\omega, z_{2y}^\omega) \\
& = \max\{\alpha_2 \ln(s_{2x}^\omega) + \ln(z_{2x}^\omega),\ \beta_2 \ln(s_{2y}^\omega) + \ln(z_{2y}^\omega)\} \\
\text{s.t.} \quad & p_x s_{2x}^\omega + p_y s_{2y}^\omega + z_{2x}^\omega + z_{2y}^\omega + \ulcorner \frac{s_{2y}^\omega}{s_{2x}^\omega+s_{2y}^\omega} \urcorner t \leq Y, \\
& s_{2k}^\omega\, s_{2l}^\omega = 0,\ s_{2k}^\omega\, z_{2l}^\omega = 0,\ z_{2k}^\omega\, z_{2l}^\omega = 0, \\
& s_{2k}^\omega, z_{2k}^\omega \geq 0, \quad \forall k, l = x, y,\ k \neq l.
\end{aligned}$$

Given a price $p = (p_x^\omega, p_y^\omega)$, the information that it conveys to all agents is denoted by $\sigma(p)$, the sub-σ-field of $\mathcal{F}$ generated by the vector-valued random variable p. Denoting $\Psi^* \equiv (\psi_x^*, \psi_y^*) = ((\Psi_x^*, 1), (\Psi_y^*, 1))$, $\varphi_j^{\omega*} \equiv (\varphi_{jx}^{\omega*}, \varphi_{jy}^{\omega*}) = ((s_{jx}^{\omega*}, z_{jx}^{\omega*}), (s_{jy}^{\omega*}, z_{jy}^{\omega*}))$, $j = 1, 2$, $\varphi_L^{\omega*} \equiv z_L^{\omega*}$, letting consumers have the same state-independent endowments $e^\omega \equiv Y$ and the landlord own $(\bar{x}, \bar{y})$, and letting μ denote a countably additive probability measure defined on $(\Omega, \mathcal{F})$, following Allen (1981), the concept of rational expectations equilibrium is formally defined as follows.

[6]The ceiling function, denoted by $\ulcorner \theta \urcorner$, is defined by the smallest integer greater than or equal to θ, i.e., $\ulcorner \theta \urcorner \equiv \min\{n \in \mathbb{Z} | \theta \leq n\}$. Notice that $\ulcorner \frac{s_{1y}}{s_{1x}+s_{1y}} \urcorner$ can be either 0 or 1, depending on whether household 1 lives in city x or y.

Definition 1 *A rational expectations equilibrium is defined as an equivalence class of $\mathcal{F}$-measurable house price functions $\Psi^* : \Omega \to \mathbb{R}^2_+$, and for each $j = 1, 2$, an equivalence class of $\mathcal{F}_j \vee \sigma(\Psi^*)$-measurable allocation functions $\varphi^*_j : \Omega \to \mathbb{R}^2_+ \cup \mathbb{R}^2_+$ such that*
(i) $\psi^{\omega *}_k \cdot \varphi^{\omega *}_{jk} \leq Y - T_k$ for μ-almost every $\omega \in \Omega$;
(ii) If $\varphi'_{jk} : \Omega \to \mathbb{R}^2_+$ satisfies the informational constraint that φ'_{jk} is $\mathcal{F}_j \vee \sigma(\Psi^*)$-measurable and the budget constraint that $\psi^{\omega *}_k \varphi^{\omega \prime}_{jk} \leq Y - T_k$ for μ-almost every $\omega \in \Omega$, then

$$\int_\Omega u^\omega_j(\varphi'_j) \, d\mu(\omega|\mathcal{F}_j \vee \sigma(\Psi^*)) \leq \int_\Omega u^\omega_j(\varphi^*_j) \, d\mu(\omega|\mathcal{F}_j \vee \sigma(\Psi^*)), \forall j = 1, 2, L;$$

(iii) $\sum_{j=1}^2 s^{\omega *}_{jk} = \bar{k}$, $\sum_{j=1}^2 z^{\omega *}_{jx} + \sum_{j=1}^2 z^{\omega *}_{jy} + z^{\omega *}_L + \ulcorner \frac{s^\omega_{1y}}{s^\omega_{1x} + s^\omega_{1y}} \urcorner t + \ulcorner \frac{s^\omega_{2y}}{s^\omega_{2x} + s^\omega_{2y}} \urcorner t = 2Y$, and $s^{\omega *}_{jk} s^{\omega *}_{jl} = 0$, $s^{\omega *}_{jk} z^{\omega *}_{jl} = 0$, $z^{\omega *}_{jk} z^{\omega *}_{jl} = 0$, $\forall k, l = x, y$, $k \neq l$, for μ-almost every $\omega \in \Omega$.

It can be seen that $\int_\Omega u^\omega_j(\varphi_j) \, d\mu(\omega|\mathcal{F}_j \vee \sigma(\Psi^*))$ is household j's expected utility of choosing φ_j, based on private information and the information given by Ψ^*.

This is the minimal perturbation of the standard general equilibrium model necessary to make it compatible with urban economics, i.e., it is the standard general equilibrium model that restricts each consumer to own housing in one and only one location. In what follows, we will solve for a bid rent equilibrium, that is equivalent to the solution of a standard market equilibrium. This device is common in urban economics, and is used by many papers.

Definition 2 *Denoting $\Psi^\omega_{jk} \equiv \max_{s^\omega_{jk}, z^\omega_{jk}} \{ \frac{Y - T_k - z^\omega_{jk}}{s^\omega_{jk}} | E[u_j|\mathcal{F}_j] = u \}$, for μ-a.e. $\omega \in \Omega$, a bid rent equilibrium is defined by $(\Psi^{\omega *}, \varphi^{\omega *}_1, \varphi^{\omega *}_2)$ such that for*

μ-almost every $\omega \in \Omega$,

$$\Psi_k^{\omega *}(u) = \max_j \{\Psi_{jk}^{\omega}(u)\}; \tag{1}$$

$$\varphi_{jk}^{\omega *} = \begin{cases} \arg\max_{s_{jk}^{\omega}, z_{jk}^{\omega}} \{\frac{Y - T_k - z_{jk}^{\omega}}{s_{jk}^{\omega}} | E[u_j|\mathcal{F}_j] = u\}, & \textit{if } j \in \arg\max_j \{\Psi_{jk}^{\omega}\}, \\ (0,0), & \textit{if } j \notin \arg\max_j \{\Psi_{jk}^{\omega}\}; \end{cases} \tag{2}$$

$$\sum_{j=1}^{2} s_{jk}^{\omega *} = \bar{k}, \quad \sum_{j=1}^{2} z_{jx}^{\omega *} + \sum_{j=1}^{2} z_{jy}^{\omega *} + z_L^{\omega *} + \ulcorner \frac{s_{1y}^{\omega}}{s_{1x}^{\omega} + s_{1y}^{\omega}} \urcorner t + \ulcorner \frac{s_{2y}^{\omega}}{s_{2x}^{\omega} + s_{2y}^{\omega}} \urcorner t = 2Y, \tag{3}$$

$$s_{jk}^{\omega *} s_{jl}^{\omega *} = 0, s_{jk}^{\omega *} z_{jl}^{\omega *} = 0, z_{jk}^{\omega *} z_{jl}^{\omega *} = 0, \ \ \forall k, l = x, y, \ k \neq l. \tag{4}$$

Since each household can consume housing in at most one city, the consumption set is $\mathbb{R}_+^2 \cup \mathbb{R}_+^2$, and the ex post state-dependent preferences of living in city k, $k = x, y$, can be specified by utilities $u_{jk} : \Omega_k \to K_{jk}$, where K_{jk} is a compact subset of $C^r(\mathbb{R}_+^2, \mathbb{R})$, $r \geq 2$, endowed with the weak C^r compact-open topology. Assume that for every ω_k, $u_{jk}^{\omega_k} \in K_{jk}$ satisfies for each $\varphi_{jk} \in \mathbb{R}_+^2$:

(i) strict (differentiable) monotonicity: $D_{\varphi} u_{jk}^{\omega_k}(\varphi) \in \mathbb{R}_{++}$,

(ii) strict (differentiable) concavity: $D_{\varphi\varphi} u_{jk}^{\omega_k}(\varphi)$ is negative definite, and

(iii) smooth boundary condition: the closure in $\mathbb{R}^2$ of the upper contour set $\{\varphi' \in \mathbb{R}_{++}^2 | u_{jk}^{\omega_k}(\varphi') \geq u_{jk}^{\omega_k}(\varphi)\}$ is contained in $\mathbb{R}_{++}^2$.

These conditions ensure that every household's state-dependent preferences are smooth in the sense of Debreu (1972) for almost every state so that, conditional on any measurable $\omega \in \mathcal{F}$ and location, demands are well defined C^{r-1} functions. Our examples satisfy these assumptions.

Although it is well-known that bid-rent and competitive equilibrium are closely connected (see for example Fujita, 1989), known results cover only the context of no uncertainty. If the rational expectations equilibria were known to be fully revealing, this result could be applied state by state. We require an equivalence result in the context of uncertainty, especially when the rational expectations equilibrium might not be fully revealing.

Lemma 1 *Given that all households' preferences are representable by a utility function satisfying conditions (i), (ii), and (iii), a triple* $(\Psi^{\omega*}, \varphi_1^{\omega*}, \varphi_2^{\omega*})$ *constitutes a bid rent equilibrium if and only if it constitutes a rational expectations equilibrium.*

Proof. See Appendix A.

2.2 Example 1

Suppose that household 1 prefers city x more than household 2, and household 2 prefers y more than household 1, i.e., $E[\alpha_1^\omega] > \alpha_2$ and $E[\beta_1^\omega] < \beta_2$.

In urban economics, as studied by Alonso (1964), bid rent describes a particular household's willingness to pay for housing in terms of composite commodity, given a fixed utility level. Following Fujita (1989), people live where their bid rents are maximal in equilibrium, and these bid rents are equilibrium rents. The bid rent functions of the two households for the housing in x and y are

$$\Psi_{1x}^\omega = \max_{s_{1x}} \frac{Y - e^{Eu_1}(s_{1x})^{-E[\alpha_1^\omega]}}{s_{1x}}, \tag{5}$$

$$\Psi_{1y}^\omega = \max_{s_{1y}} \frac{Y - t - e^{Eu_1}(s_{1y})^{-E[\beta_1^\omega]}}{s_{1y}}, \tag{6}$$

$$\Psi_{2x}^\omega = \max_{s_{2x}^\omega} \frac{Y - e^{u_2^\omega}(s_{2x}^\omega)^{-\alpha_2}}{s_{2x}^\omega}, \tag{7}$$

$$\Psi_{2y}^\omega = \max_{s_{2y}^\omega} \frac{Y - t - e^{u_2^\omega}(s_{2y}^\omega)^{-\beta_2}}{s_{2y}^\omega}, \tag{8}$$

where $\omega \in \Omega$. From first and second-order conditions, the optimal land lot sizes for households are

$$s_{1x}^{\omega*} = [\frac{e^{Eu_1}(1+E[\alpha_1^\omega])}{Y}]^{\frac{1}{E[\alpha_1^\omega]}}, \tag{9}$$

$$s_{1y}^{\omega*} = [\frac{e^{Eu_1}(1+E[\beta_1^\omega])}{Y-t}]^{\frac{1}{E[\beta_1^\omega]}}, \tag{10}$$

$$s_{2x}^{\omega*} = [\frac{e^{u_2^\omega}(1+\alpha_2)}{Y}]^{\frac{1}{\alpha_2}}, \tag{11}$$

$$s_{2y}^{\omega*} = [\frac{e^{u_2^\omega}(1+\beta_2)}{Y-t}]^{\frac{1}{\beta_2}}. \tag{12}$$

Recall that land sizes of city x and city y are $\bar{x}$ and $\bar{y}$, respectively. From market clearing conditions $s_{jx}^{\omega*} = \bar{x}$ and $s_{jy}^{\omega*} = \bar{y}$, we have

$$Eu_1^* = \begin{cases} \ln[Y] + E[\alpha_1^\omega]\ln[\bar{x}] - \ln[1+E[\alpha_1^\omega]], & \text{if household 1 lives at } x; \\ \ln[Y-t] + E[\beta_1^\omega]\ln[\bar{y}] - \ln[1+E[\beta_1^\omega]], & \text{if household 1 lives at } y, \end{cases} \tag{13}$$

$$u_2^{\omega*} = \begin{cases} \ln[Y] + \alpha_2\ln[\bar{x}] - \ln[1+\alpha_2], & \text{if household 2 lives at } x; \\ \ln[Y-t] + \beta_2\ln[\bar{y}] - \ln[1+\beta_2], & \text{if household 2 lives at } y, \end{cases} \tag{14}$$

for $\omega \in \Omega$. So the equilibrium bid rents of agents in the two cities in two states are

$$\Psi_{1x}^{\omega*} = \frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]}\frac{Y}{\bar{x}}, \tag{15}$$

$$\Psi_{1y}^{\omega*} = \frac{E[\beta_1^\omega]}{1+E[\beta_1^\omega]}\frac{Y-t}{\bar{y}}, \tag{16}$$

$$\Psi_{2x}^{\omega*} = \frac{\alpha_2}{1+\alpha_2}\frac{Y}{\bar{x}}, \tag{17}$$

$$\Psi_{2y}^{\omega*} = \frac{\beta_2}{1+\beta_2}\frac{Y-t}{\bar{y}}, \tag{18}$$

for $\omega \in \Omega$. The equilibrium bid rents are presented in Figure 1, where the horizontal axis represents the amount of transportation cost and the vertical axis represents the individual bid rents.

Since $\frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]} > \frac{\alpha_2}{1+\alpha_2}$ if and only if $E[\alpha_1^\omega] > \alpha_2$, given $E[\alpha_1^\omega] > \alpha_2$, the bid rent of household 1 for the housing in x is higher than that of household

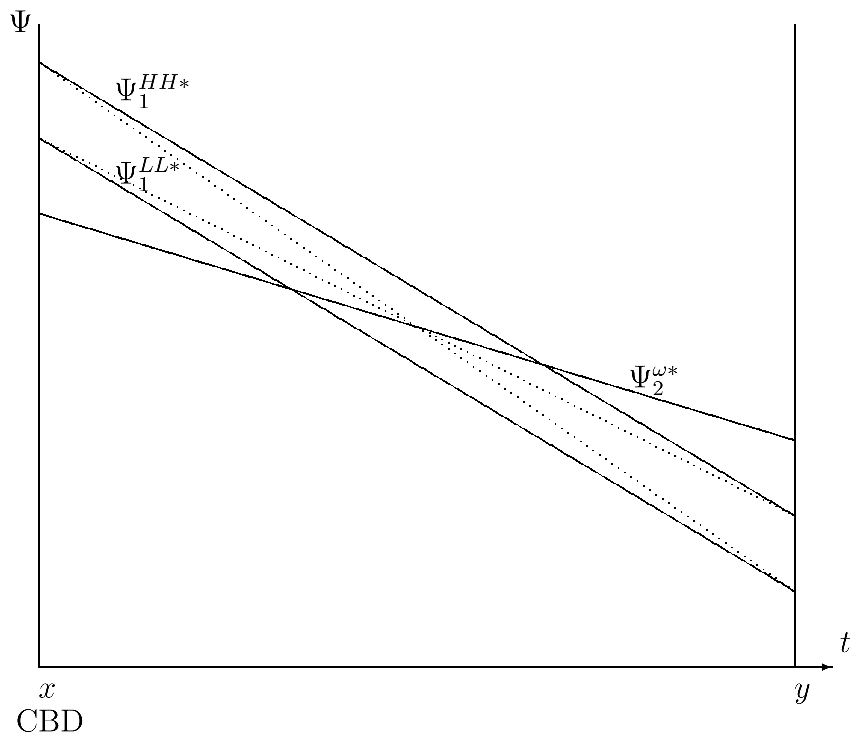

Figure 1: The bid rent functions in Example 1.

2 for the housing in x in both states. Similarly, since $\frac{E[\beta_1^\omega]}{1+E[\beta_1^\omega]} < \frac{\beta_2}{1+\beta_2}$ if and only if $E[\beta_1^\omega] < \beta_2$, $E[\beta_1^\omega] < \beta_2$ implies that the bid rent of household 1 for the housing in y is lower than that of household 2 for the housing in y in all states. Therefore, the location pattern is verified under the conditions we have assumed.

Notice that there is no equilibrium that fully reveals information. If in equilibrium $\Psi_x^{HH*} = \Psi_x^{HL*} \neq \Psi_x^{LH*} = \Psi_x^{LL*}$, either the valuation of household 1 for the housing in city x differs in different states (in city x), which conflicts with the assumption that household 1 has no information about the state, or the equilibrium bid rent is determined by household 2's valuation, which conflicts with the assumption $E[\beta_1^\omega] < \beta_2$. Notice also that $\Psi_x^{\omega *}$ and $\Psi_y^{\omega *}$ depend only on the mean of α_1, β_2, and the values of Y, t, $\bar{x}$, and $\bar{y}$. Therefore, the equilibrium rents in the two cities are independent of the realized state.

2.3 Example 2

Follow the same idea as in the previous example, but suppose that household 1 now knows the state in city y, but has no information about city x. On the other hand, household 2 knows only the state in city x, but not the state in y. Furthermore, the states in the two cities are not correlated. That is, let $\Omega \equiv \Omega_x \times \Omega_y$, where $\Omega_x = \Omega_y \equiv \{H, L\}$ represent the state spaces in cities x and y. $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y, \{H\}, \{L\}\}$, $\mathcal{F}_2 = \{\phi, \Omega_x, \{H\}, \{L\}\} \times \{\phi, \Omega_y\} \subseteq \mathcal{F}$ are sub-σ-fields representing private information. Again, the relationship between states and prices are common knowledge.

Each household chooses to live in one and only one city. Moreover, households make their decisions simultaneously. Given an event $\omega \in \Omega$, both households' utilities are state-dependent, so their optimization problems are

$$\begin{aligned}
\max_{s_{1x}, s_{1y}^{\omega}, z_{1x}, z_{1y}^{\omega}} \quad & Eu_1(s_{1x}, s_{1y}^{\omega}, z_{1x}, z_{1y}^{\omega} | \mathcal{F}_1) \\
& = \max\{E[\alpha_1^{\omega} \ln(s_{1x}) + \ln(z_{1x}) | \mathcal{F}_1],\ \beta_1^{\omega} \ln(s_{1y}^{\omega}) + \ln(z_{1y}^{\omega})\} \\
\text{s.t.} \quad & p_x s_{1x} + p_y s_{1y}^{\omega} + z_{1x} + z_{1y}^{\omega} + \ulcorner \frac{s_{1y}^{\omega}}{s_{1x}^{\omega} + s_{1y}^{\omega}} \urcorner\, t \leq Y, \\
& s_{1x}\, s_{1y}^{\omega} = 0,\ s_{1x}\, z_{1y}^{\omega} = 0,\ z_{1x}\, s_{1y}^{\omega} = 0,\ z_{1x}\, z_{1y}^{\omega} = 0, \\
& s_{1x}, s_{1y}^{\omega}, z_{1x}, z_{1y}^{\omega} \geq 0;
\end{aligned}$$

$$\begin{aligned}
\max_{s_{2x}^{\omega}, s_{2y}, z_{2x}^{\omega}, z_{2y}} \quad & Eu_2(s_{2x}^{\omega}, s_{2y}, z_{2x}^{\omega}, z_{2y} | \mathcal{F}_2) \\
& = \max\{\alpha_2^{\omega} \ln(s_{2x}^{\omega}) + \ln(z_{2x}^{\omega}),\ E[\beta_2^{\omega} \ln(s_{2y}) + \ln(z_{2y}) | \mathcal{F}_2]\} \\
\text{s.t.} \quad & p_x s_{2x}^{\omega} + p_y s_{2y} + z_{2x}^{\omega} + z_{2y} + \ulcorner \frac{s_{2y}^{\omega}}{s_{2x}^{\omega} + s_{2y}^{\omega}} \urcorner\, t \leq Y, \\
& s_{2x}^{\omega}\, s_{2y} = 0,\ s_{2x}^{\omega}\, z_{2y} = 0,\ z_{2x}^{\omega}\, s_{2y} = 0,\ z_{2x}^{\omega}\, z_{2y} = 0, \\
& s_{2x}^{\omega}, s_{2y}, z_{2x}^{\omega}, z_{2y} \geq 0;
\end{aligned}$$

Note that in fact, the optimized utility of household 1 is state-dependent (state-independent) at y (x), denoted by $u_1^{\omega *}$ (Eu_1^*); $u_2^{\omega *}$ and Eu_2^* are similarly defined. To present an example of rational expectations equilibrium without revealing private information, suppose that $E[\alpha_1^\omega] > \alpha_2^\omega$ and $E[\beta_2^\omega] > \beta_1^\omega$, for all $\omega \in \Omega$.

Given these conditions, suppose that households 1 and 2 choose to live in cities x and y, respectively. Their bid rent functions are, $\forall \omega \in \Omega$,

$$\Psi_{1x}^\omega = \max_{s_{1x}} \frac{Y - e^{Eu_1} s_{1x}^{-E[\alpha_1^\omega]}}{s_{1x}}, \tag{19}$$

$$\Psi_{1y}^\omega = \max_{s_{1y}} \frac{Y - t - e^{u_1^\omega} s_{1y}^{-\beta_1^\omega}}{s_{1y}}, \tag{20}$$

$$\Psi_{2x}^\omega = \max_{s_{2x}} \frac{Y - e^{u_2^\omega} s_{2x}^{-\alpha_2^\omega}}{s_{2x}}, \tag{21}$$

$$\Psi_{2y}^\omega = \max_{s_{2y}} \frac{Y - t - e^{Eu_2} s_{2y}^{-E[\beta_2^\omega]}}{s_{2y}}. \tag{22}$$

Thus, the optimal land sizes for household 1 and 2 are, $\forall \omega \in \Omega$,

$$s_{1x}^{\omega *} = [\frac{e^{Eu_1}(1 + E[\alpha_1^\omega])}{Y}]^{\frac{1}{E[\alpha_1^\omega]}}, \tag{23}$$

$$s_{1y}^{\omega *} = [\frac{e^{u_1^\omega}(1 + \beta_1^\omega)}{Y - t}]^{\frac{1}{\beta_1^\omega}}, \tag{24}$$

$$s_{2x}^{\omega *} = [\frac{e^{u_2^\omega}(1 + \alpha_2^\omega)}{Y}]^{\frac{1}{\alpha_2^\omega}}, \tag{25}$$

$$s_{2y}^{\omega *} = [\frac{e^{Eu_2}(1 + E[\beta_2^\omega])}{Y - t}]^{\frac{1}{E[\beta_2^\omega]}}. \tag{26}$$

From $s_{jx}^{\omega *} = \bar{x}$ and $s_{jy}^{\omega *} = \bar{y}$, we have

$$Eu_1^*(\cdot|\mathcal{F}_1) = \begin{cases} \ln[Y] + E[\alpha_1^\omega]\ln[\bar{x}] - \ln[1 + E[\alpha_1^\omega]], & \text{if household 1 lives at } x; \\ \ln[Y - t] + \beta_1^\omega \ln[\bar{y}] - \ln[1 + \beta_1^\omega], & \text{if household 1 lives at } y, \end{cases} \tag{27}$$

$$Eu_2^*(\cdot|\mathcal{F}_2) = \begin{cases} \ln[Y] + \alpha_2^\omega \ln[\bar{x}] - \ln[1 + \alpha_2^\omega], & \text{if household 2 lives at } x; \\ \ln[Y - t] + E[\beta_2^\omega]\ln[\bar{y}] - \ln[1 + E[\beta_2^\omega]], & \text{if household 2 lives at } y. \end{cases} \tag{28}$$

Again, agents' equilibrium bid rents are

$$\Psi_{1x}^{\omega *} = \frac{E[\alpha_1^\omega]}{1 + E[\alpha_1^\omega]} \frac{Y}{\bar{x}}, \tag{29}$$

$$\Psi_{1y}^{\omega *} = \frac{\beta_1^\omega}{1 + \beta_1^\omega} \frac{Y - t}{\bar{y}}, \tag{30}$$

$$\Psi_{2x}^{\omega *} = \frac{\alpha_2^\omega}{1 + \alpha_2^\omega} \frac{Y}{\bar{x}}, \tag{31}$$

$$\Psi_{2y}^{\omega *} = \frac{E[\beta_2^\omega]}{1 + E[\beta_2^\omega]} \frac{Y - t}{\bar{y}}, \tag{32}$$

where $\omega \in \Omega$. The equilibrium bid rents are drawn in Figure 2, where the horizontal axis represents the transportation cost and the individual bid rents are represented by vertical axis.

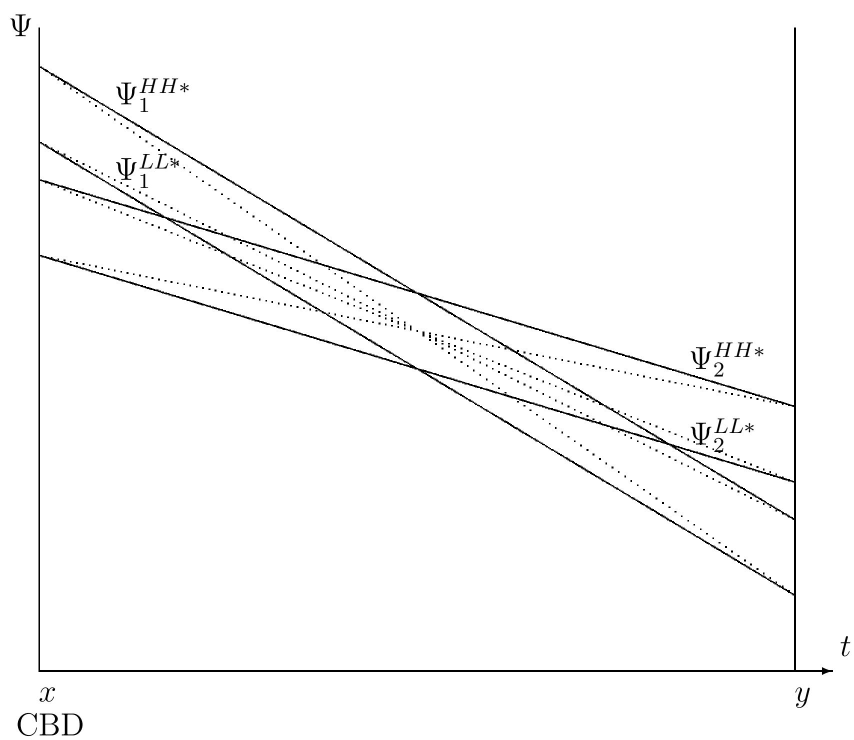

Figure 2: The bid rent functions in Example 2.

Inequalities $E[\alpha_1^\omega] > \alpha_2^\omega$ and $E[\beta_2^\omega] > \beta_1^\omega$, $\forall \omega$, imply that the bid rent of household 1 (household 2) for the housing in x (y) is always higher than that of household 2 (household 1). So the equilibrium location pattern where household 1 lives at x and household 2 lives at y is verified.[7]

Again, there is no fully revealing equilibrium in this example. Since Ψ_x^* and Ψ_y^* depend only on Y, t, the mean of the preference parameters and the sizes of land in cities, the equilibrium rents are constants in all the realized states. That is, the mapping from prices to preferences is not injective, which is the source of the non-existence of fully-revealing rational expectations equilibrium.

3 An Open Subset of Economies without Fully Revealing Equilibria

From the two examples, the existence of two points in the parameter space with no fully revealing rational expectations equilibrium is shown. In this section, we generalize the examples and show that, in economies under uncertainty where there is no market for contingency claims contracts, the fully revealing rational expectations equilibria cannot exist for an open set of utility functions. But for all parameters satisfying specific conditions, there exists a rational expectations equilibrium that is not fully revealing, which will be proved in the next section.[8]

Suppose there are two households ($j = 1, 2$), one landlord ($j = L$), and two cities ($k = x, y$). Let $\Omega \equiv \Omega_x \times \Omega_y$ be a finite payoff-relevant state space of the economy, and every element $\omega \equiv \omega_x \times \omega_y \in \Omega$ is termed as a state

[7]Even when households can observe other households' consumptions (of housing and composite good), given that the states in two cities are not correlated, the non-existence of fully-revealing generalized rational expectations equilibria (GREE) still holds in this example.

[8]These conditions are in fact the ordered relative steepness and the local information conditions.

of the economy. Every household is endowed with the same initial state-independent endowment Y, a private information sub-σ-field $\mathcal{F}_j \subsetneqq \mathcal{F}$, and a state-dependent utility function $u_j \equiv u_{jx} \vee u_{jy}$ defined on $\Omega \times (\mathbb{R}^2_+ \cup \mathbb{R}^2_+)$ which means that households must each choose a location. Households are assumed to maximize their conditional expected utilities, where the ex post state-dependent preferences of living in city k are specified by $u_{jk} : \Omega_k \to K_{jk}$, where K_{jk} is a compact subset of $C^r(\mathbb{R}^2_+, \mathbb{R})$ functions, $r \geq 2$, which is endowed with the weak C^r compact-open topology. For each state of information ω, the economy $(Y, u^\omega_j(\cdot))^n_{j=1}$ is a smooth economy as defined by Debreu (1972). It is important to notice that u_{jk} is payoff-relevant to only Ω_k, that is, we assume that people living in city k cares only the state in k. Later, we consider the perturbations with this property being maintained.

The general optimization problem for household j with two cities, given his/her information structure $\mathcal{F}_j$, is:

$$\begin{aligned} \max_{s^\omega_{jx}, s^\omega_{jy}, z^\omega_{jx}, z^\omega_{jy}} \quad & Eu_j(s^\omega_{jx}, s^\omega_{jy}, z^\omega_{jx}, z^\omega_{jy} | \mathcal{F}_j) \\ \text{s.t.} \quad & p_x s^\omega_{jx} + p_y s^\omega_{jy} + z^\omega_{jx} + z^\omega_{jy} + \ulcorner \frac{s^\omega_{jy}}{s^\omega_{jx} + s^\omega_{jy}} \urcorner \, t \leq Y, \\ & s^\omega_{jk} \, s^\omega_{jl} = 0, \; s^\omega_{jk} \, z^\omega_{jl} = 0, \; z^\omega_{jk} \, z^\omega_{jl} = 0, \\ & s^\omega_{jk}, z^\omega_{jk} \geq 0, \;\; \forall k, l = x, y, \; k \neq l, \\ & (s^\omega_{jx}, s^\omega_{jy}, z^\omega_{jx}, z^\omega_{jy}) \text{ is } \mathcal{F}_j\text{-measurable.} \end{aligned} \tag{33}$$

Before we prove the results, some equilibrium concepts must be defined. In a rational expectations equilibrium, the information can be fully revealing, which means that all households can learn the state of nature by observing the equilibrium price and using their private information. Alternatively, the information can be non-fully revealing in a rational expectations equilibrium, where at least one household cannot tell the state of nature from the equilibrium price and their private information. Their mathematical definitions are as follows:

Definition 3 *A fully-revealing rational expectations equilibrium is a rational expectations equilibrium such that*

$$\mathcal{F}_j \vee \sigma(\Psi^*) = \mathcal{F}, \quad \forall j \in \{1, 2\}. \tag{34}$$

When there is at least one j such that the above equality does not hold, we say it is a non-fully-revealing rational expectations equilibrium.

In other words, conditioning on a fully revealing equilibrium price function is equivalent to knowing the pooled information of all households in the economy. Though Allen (1981) proves the existence of an open and dense subset of economies with fully-revealing rational expectations equilibrium in the classical framework, when perturbations location-by-location are considered, Theorem 1 shows that the same statement does not hold in urban economics. Perturbations location-by-location are defined formally as local perturbations as follows.[9]

Definition 4 (Local Perturbations)
The perturbations of households' preferences are named as local perturbations when they satisfy that u_{jk} after perturbations is independent of $\Omega_{k'}$, $\forall j = 1, 2$, $\forall k, k' = 1, 2$, $k' \neq k$.

In other words, local perturbations require that each household's utility in k is measurable with respect to only Ω_k. Local perturbations is equivalent to saying that people living in a city care only about the state in the city they live. Local perturbations are more realistic than non-local perturbations in urban economics, since it is not persuasive to say that the perturbed preferences in city k depends on the states in another city. For example,

[9] Throughout this paper, only the preference perturbations are considered since endowment perturbations must give households more information from budget balance binding, and information perturbations are not smooth.

when preference perturbations are considered, in most cases, the utility of living in Chicago is irrelevant to the circumstances in New York. Therefore, in urban economics, it doesn't make sense to consider non-local perturbations as used in standard models. Throughout this paper, to highlight the distinct essences in urban economics, we focus on local perturbations.

Theorem 1 *Given a discrete state space Ω with at least 2 states, consider local perturbations of households' preferences, there exists an open subset of economies that possess no fully-revealing rational expectations equilibrium.*

Proof. Consider example 1 first. Notice that in equilibrium, household 1's marginal rate of substitution for housing in city x is $\frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]}\frac{Y}{\bar{x}}$. On the other hand, household 2's marginal rate of substitution for housing in x is $\frac{\alpha_2^\omega}{1+\alpha_2^\omega}\frac{Y}{\bar{x}}$. Let $\alpha_1^{HH} = \alpha_1^{HL} > \alpha_1^{LH} = \alpha_1^{LL}$ and $\beta_1^{HH} = \beta_1^{LH} > \beta_1^{HL} = \beta_1^{LL}$.

Since in the example $E[\alpha_1^\omega] > \alpha_2$ and $E[\beta_1^\omega] < \beta_2$, we can choose $\epsilon^\alpha = \frac{E[\alpha_1^\omega]-\alpha_2}{(E[\alpha_1^\omega]+\alpha_2)Y+(2+E[\alpha_1^\omega]+\alpha_2)\bar{x}} > 0$, $\epsilon^\beta = \frac{\beta_2-E[\beta_1^\omega]}{(E[\beta_1^\omega]+\beta_2)Y+(2+E[\beta_1^\omega]+\beta_2)\bar{y}} > 0$, and $\epsilon = \epsilon^\alpha \wedge \epsilon^\beta$. Recall that the equilibrium marginal utilities in example 1 are

$$v^* \equiv (D_{s_{1x}}Eu_1^*, D_{s_{1y}}Eu_1^*, D_{z_{1x}}Eu_1^*, D_{z_{1y}}Eu_1^*, D_{s_{2x}}u_2^{\omega *}, D_{s_{2y}}u_2^{\omega *}, D_{z_{2x}}u_2^{\omega *}, D_{z_{2y}}u_2^{\omega *}).$$

Centered at v^*, consider all perturbations of utility functions within an open set in the weak C^r topology such that

$$D_{s_{1k}}Eu_1 \in (D_{s_{1k}}Eu_1^* - \epsilon, D_{s_{1k}}Eu_1^* + \epsilon), \tag{35}$$

$$D_{z_{1k}}Eu_1 \in (D_{z_{1k}}Eu_1^* - \epsilon, D_{z_{1k}}Eu_1^* + \epsilon), \tag{36}$$

$$D_{s_{2k}}u_2^\omega \in (D_{s_{2k}}u_2^{\omega *} - \epsilon, D_{s_{2k}}u_2^{\omega *} + \epsilon), \tag{37}$$

$$D_{z_{2k}}u_2^\omega \in (D_{z_{2k}}u_2^{\omega *} - \epsilon, D_{z_{2k}}u_2^{\omega *} + \epsilon), \qquad k = x, y. \tag{38}$$

These perturbations are evaluated at city k, $k = x, y$, individually, and are thus local perturbations. Then it can be checked that all utilities within this neighborhood generate bid rents that are within ϵ of the equilibrium bid rents in example 1. Furthermore, household 1's realized marginal rate of substitution for housing in city x is always higher than the marginal rate of substitution of household 2; household 2's marginal rate of substitution for housing in city y is always higher than that of household 1.[10]

Now we can prove the non-existence of fully revealing rational expectations equilibrium. Suppose for any set of preferences within these local perturbations, there exists a fully revealing rational expectations equilibrium $(\varphi_1^*, \varphi_2^*, \Psi^*)$. Then the uninformed household (household 1) can infer the state of nature by observing Ψ^*. However, within the perturbations, the equilibrium bid rents are the same across states, contradicting that Ψ^* is a fully-revealing rational expectations equilibrium price.

Obviously, a similar argument works for the cases with more than 2 states and example 2. *Q.E.D.*

Conventional wisdom says that if one household doesn't live in a specific city, he/she doesn't have the information about that city. However, this paper shows that even if one household has the information about a specific city, if he/she doesn't live there, the housing price in that city cannot reveal his/her information. The arrangement of a household living in the city that he/she is informed yields not only a information gain for himself/herself (that he/she can maximize ex post utility instead of expected utility), but also a information spillover to all other households that they can learn the state of that city by observing equilibrium housing price. When local perturbations are considered, the information spillover plays no role for the households living in other cities. However, when non-local perturbations are considered,

[10]In city x, for example, since the lowest MRS for household 1 is $\frac{E[\alpha_1^\omega]/\bar{x}-\epsilon}{(1+E[\alpha_1^\omega])/Y+\epsilon}$, and the highest MRS for household 2 is $\frac{\alpha_2/\bar{x}+\epsilon}{(1+\alpha_2)/Y-\epsilon}$, household 1's MRS is greater than household 2's MRS if and only if $\epsilon < \epsilon^\alpha = \frac{E[\alpha_1^\omega]-\alpha_2}{(E[\alpha_1^\omega]+\alpha_2)Y+(2+E[\alpha_1^\omega]+\alpha_2)\bar{x}}$. Similarly, household 2's MRS in city y is greater than that of household 1 if and only if $\epsilon < \epsilon^\beta = \frac{\beta_2-E[\beta_1^\omega]}{(E[\beta_1^\omega]+\beta_2)Y+(2+E[\beta_1^\omega]+\beta_2)\bar{y}}$.

a small perturbation makes the utility in city k be relevant to the states of all cities, then as shown in Allen (1981), generically there exists an open and dense set of economies possessing fully revealing rational expectations equilibrium.

Finally, we make a remark here: If there is no fully revealing rational expectations equilibrium, an equilibrium allocation can fail to be a Pareto optimum. Consider a variation of Example 1 shown in Figure 3. When the probability is quite evenly distributed over Ω_k, $k = 1, 2$, household 1's bid rent for city 1 is larger than that of household 2, and household 2's bid rent for city 2 is larger than that of household 1. So in equilibrium, household j lives in city j, $j = 1, 2$ in both states. However, in a Pareto optimum, household j lives in city $3 - j$, $j = 1, 2$ when $\omega = LL$. Therefore, here we get an example with an equilibrium allocation that is ex ante but not ex post efficient.

4 The Existence of Rational Expectations Equilibrium

After presenting an open subset of economies that possess non-fully-revealing rational expectations equilibrium, it is natural to ask: Can a rational expectations equilibrium fail to exist in urban economics? This can undermine the minimal requirement for further analysis in urban economics with uncertainty. In this section, the existence of (not necessarily fully-revealing) rational expectations equilibrium is examined, given the assumption of ordered relative steepness of bid-rents. First we describe how the existence of equilibrium depends on the number of locations relative to the number of

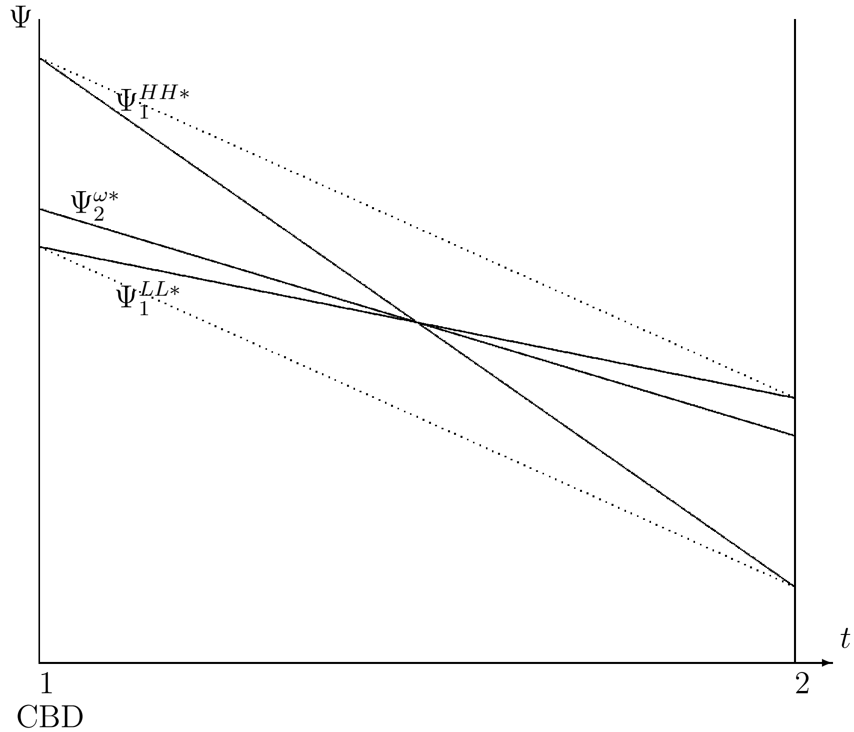

Figure 3: The non-fully revealing rational expectations equilibrium can fail to be Pareto optimal.

households.[11]

When the number of locations is greater than the number of households, since each household can consume housing in at most one city, there must exist at least one city where no household lives. In these abandoned cities, by Walras' Law, the price of housing is zero. Therefore, unless the transportation cost is very high (or these cities are very far away from inhabited cities), households have an incentive to move into these cities to enjoy a higher utility.

When the number of locations is the same as the number of households, the assumption of ordered relative steepness of bid rents ensures that every

[11]By contrast, in standard models, the focus is on the dimension of prices (which is the same as the number of cities in our examples) relative to the dimension of parameters, where the dimension of discrete state space is 0.

location is occupied by exactly one household in equilibrium. Therefore, we can settle households one-by-one from CBD to periphery in the order of the slopes of their bid rents, constituting an equilibrium allocation.[12] Thus, we know ex ante what information will be revealed by equilibrium prices, so we can add this information to the consumer's optimization problem. The case when the number of households is larger than the number of locations is left to future work. This case is difficult because we don't know ex ante where consumers will reside in equilibrium, and we don't know what information will be revealed by equilibrium prices.

Before proving a theorem on the existence of equilibrium, we need to make following assumptions on households' bid rents. These assumptions are standard in urban economics, for example, Fujita (1985, 1989).[13] Given a distance t from CBD, a specific state ω, and a utility level u, denote $\Psi_j^\omega(t,u) \equiv \max_{s_j^\omega, z_j^\omega}\{\frac{Y-t-z_j^\omega}{s_j^\omega}|u_j = u\}$ to be household j's bid rent for the housing in the distance t, given ω and u.[14]

Assumption 1 (Ordered Relative Steepness)
Households' bid rent functions are ordered by their relative steepnesses. That is, given $j < j' \leq n$, Ψ_j^ω is steeper than $\Psi_{j'}^\omega$: Whenever $\Psi_j^\omega(\bar{t}, u_j) = \Psi_{j'}^\omega(\bar{t}, u_{j'}) > 0$ for some $\bar{t}$, u_j and $u_{j'}$, then

$$\Psi_j^\omega(t, u_j) > \Psi_{j'}^\omega(t, u_{j'}) \ \ \forall\, 0 \leq t < \bar{t}, \tag{39}$$

$$\Psi_j^\omega(t, u_j) < \Psi_{j'}^\omega(t, u_{j'}) \ \ \forall\, \bar{t} < t \ \textit{and}\ \Psi_j^\omega(t, u_j) > 0. \tag{40}$$

[12]Without the assumption of ordered relative steepness of bid rents, we must find a fixed point in the information structure.

[13]In fact, the assumption of ordered relative steepness relates to only the uniqueness of equilibrium and makes the proof easier, but existence of equilibrium in urban economics can be proved without this assumption when there is no uncertainty, see Fujita and Smith (1987).

[14]Notice that though cities are discrete points on the distance line, households' bid rents are in fact continuous functions of the distance from CBD.

The assumption of ordered relative steepness ensures that given arbitrary levels of utilities for two agents, their bid rents can cross in at most one point as shown in Figure 4; In equilibrium, must cross at one and only one point. For example, Cobb-Douglas utilities in Example 1 and 2 satisfy the assumption of ordered relative steepness, and so do quasi-linear utilities. In what follows, we prove the existence of rational expectations equilibrium given the assumptions of ordered relative steepness and local information.

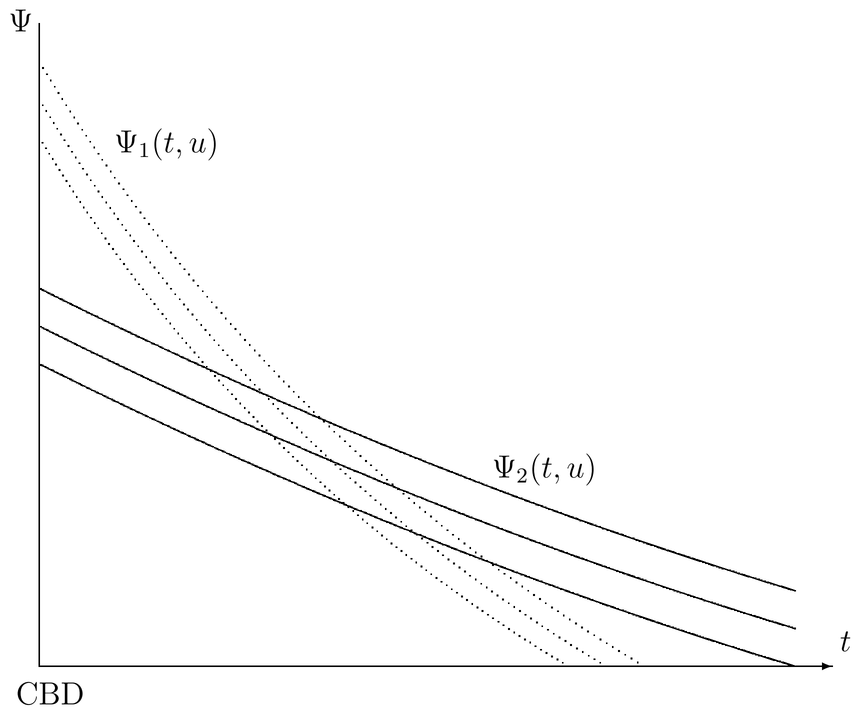

Figure 4: When households' bid rents satisfy ordered relative steepness assumption.

4.1 When households have local information

Given that there are n households indexed by $j \in N \equiv \{1, ..., n\}$, and n cities, $k \in K \equiv \{1, ..., n\}$, indexed by the order of increasing distance from the CBD. That is, recall that T_k denotes the commuting cost from city k to

CBD, $0 \leq T_1 < T_2 < ... < T_n < Y$ is assumed to ensure that there is no vacant city. Denote $\bar{s}_k$ to be the land supply in city k. Also let $\tilde{\sigma}_k \equiv \sigma(\Omega_k) \times (\times_{k' \neq k}\{\phi, \Omega_{k'}\})$, which is the σ-algebra indicating that only the state in city k is comprehended and all states in other cities are completely unknown. Since the settings of examples highlight the required condition for the existence of non-fully revealing rational expectations equilibrium, in what follows, we focus on the cases when households have local information.

Definition 5 (Local Information)
For every household j such that $\tilde{\sigma}_k \subseteq \mathcal{F}_j$, there exist $\omega_k, \omega'_k \in \Omega_k$, $\omega_k \neq \omega'_k$, such that the realized marginal rate of substitution is the same for $\omega_k, \omega'_k \in \Omega_k$, $\forall j' \neq j$, $u_{j'k}^{\omega_k} \neq u_{j'k}^{\omega'_k}$.

The notation $\tilde{\sigma}_k \subseteq \mathcal{F}_j$ says that $\mathcal{F}_j$ is finer than $\tilde{\sigma}_k$. The intuition of the assumption of local information is that for the household who owns (at least) the information of city k, his/her marginal rate of substitution in city k is independent of (at least) two realized states. However, to ensure that his/her information is not trivial, we need the second part of the assumption which implies that his/her information about city k does matter for some household else. Define $\hat{\mathcal{F}}_{jk} \equiv \mathcal{F}_j \wedge (\times_{i \neq j}\{\phi, \Omega_i\})$, that is, $\hat{\mathcal{F}}_{jk}$ is the partition indicating that only household j's information about city k is revealed, while his/her information about other cities are not revealed. Hence, $\tilde{\sigma}_k \subseteq \mathcal{F}_j$ is equivalent to $\hat{\mathcal{F}}_{jk} = \tilde{\sigma}_k$ in presenting that household j has local information about city k. Now, we can discuss the relationship between local information and the existence of equilibrium.[15]

Theorem 2 *When any information is local, given Assumption 1, under generically local perturbations, there exists a unique non-fully revealing ra-*

[15]There exists a stronger condition for the first part of Theorem 2, $E[u_{jk}|\mathcal{F}_j] = E[u_{jk}|\hat{\mathcal{F}}_{jk}]$, which is sufficient but not necessary for local information.

tional expectations equilibrium such that, for $k \in \{1, ..., n\}$,

$$\Psi_k^{\omega *}(u) = \Psi_{kk}^{\omega}(u) = \max_{s_{kk}^{\omega}, z_{kk}^{\omega}} \{ \frac{Y - T_k - z_{kk}^{\omega}}{s_{kk}^{\omega}} | E[u_{kk} | \hat{\mathcal{F}}_{kk}] = u \}; \tag{41}$$

$$\varphi_{jk}^{\omega *}(u) = \begin{cases} (\bar{s}_k, \; Y - T_k - \Psi_k^{\omega *}(u) \, \bar{s}_k), & \text{if } \; j = k, \\ (0, 0), & \text{if } \; j \neq k; \end{cases} \tag{42}$$

and the equilibrium utility level u^{**} *can be solved by*

$$\Psi_k^{\omega *}(u^{**}) = \frac{D_{s_{kk}^{\omega}} E u_{kk}}{D_{z_{kk}^{\omega}} E u_{kk}} \Big|_{\varphi_{kk}^{\omega *}(u^{**})}. \tag{43}$$

Proof. First, given Assumption 1, every city is occupied by exactly one household; otherwise, there exists an empty city with zero housing price (by Walras' Law) where all households will move. Second, by Lemma 1, the rational expectations equilibrium corresponds to the bid rent equilibrium. Since household 1 has the steepest bid rent, from equation (1) in Definition 2, he/she must occupy the housing in city 1 in equilibrium. After settling household 1, we can consider the problem as the one with $n - 1$ households ($j \in \{2, ..., n\}$) and $n - 1$ cities ($k \in \{2, ..., n\}$). Then, household 2 has a steeper bid rents than remaining households, so he/she wins the housing in city 2. Following the same logic, in equilibrium all households are arranged that household j lives in city j, or say, city k is occupied by household k, and no one has an incentive to deviate. This is a standard argument in urban economics.

As shown in Figure 5, given that household k is located in city k, the intercept of budget line $Y - T_k$ and the housing supply $\bar{s}_k$ are determined by parameters. Now, given arbitrary u, the slope of budget line $\Psi_k^{\omega *}(u)$ and thus the corresponding $\varphi_{kk}^{\omega *}(u)$ is uniquely determined (by the cross point of budget line and $\bar{s}_k$). Furthermore, given consumption point $\varphi_{kk}^{\omega *}(u)$, since households' preferences are smooth, the slope of the indifference curve passing through $\varphi_{kk}^{\omega *}$ is uniquely determined. Finally, the equilibrium utility level (and the equilibrium housing price in city k) is given by $\Psi_k^{\omega*}(u) = \frac{D_{s_{kk}^{\omega}} Eu_{kk}}{D_{z_{kk}^{\omega}} Eu_{kk}} \Big|_{\varphi_{kk}^{\omega*}(u)}$, as shown in Figure 4. Let $f(u) \equiv \Psi_k^{\omega*}(u) - \frac{D_{s_{kk}^{\omega}} Eu_{kk}}{D_{z_{kk}^{\omega}} Eu_{kk}} \Big|_{\varphi_{kk}^{\omega*}(u)}$, since $\Psi_k^{\omega*}$ and marginal rate of substitution are continuous in u, $f(u)$ is continuous in u. At $\bar{E}$, $f(u) < 0$ since $\Psi_k^{\omega*}(u) = 0$ at $\bar{E}$. Given $\bar{s}_k > 0$, by smooth boundary condition, $\frac{D_{s_{kk}^{\omega}} Eu_{kk}}{D_{z_{kk}^{\omega}} Eu_{kk}} \Big|_{\varphi_{kk}^{\omega*}(u)} \to 0$ as $z_{kk}^{\omega} \to 0$, which implies that $\exists \underline{u}$ such that $f(u) > 0, \forall u \leq \underline{u}$. Therefore, by the intermediate value theorem, there exists a u^{**} solving $f(u) = 0$ and thus there exists a rational expectations equilibrium. The uniqueness of equilibrium can be guaranteed by the condition that $\frac{D_{s_{kk}^{\omega}} Eu_{kk}}{D_{z_{kk}^{\omega}} Eu_{kk}} \Big|_{\varphi_{kk}^{\omega*}(u)}$ is increasing with u, which is true when the consumption of housing is a normal good as shown in Fujita and Berliant (1992).

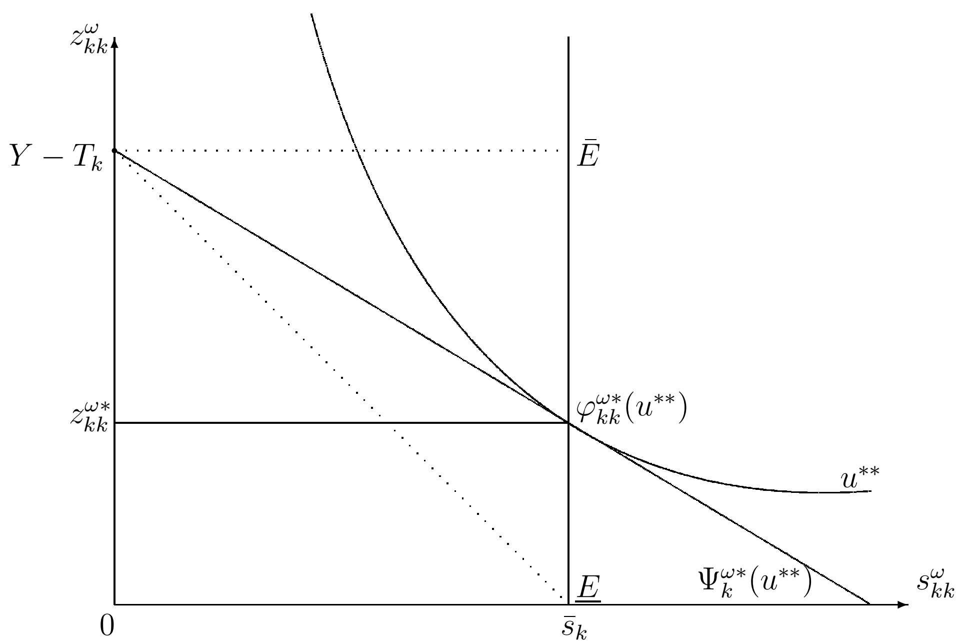

Figure 5: The determination of equilibrium housing price and equilibrium utility for household k in city k in state ω, $k \in \{1, ..., n\}$.

When every information is local, we want to prove that the unique rational expectations equilibrium is non-fully revealing. Suppose the equilibrium is fully-revealing, then choose arbitrary k, we can have

$$\Psi_k^{\omega_k *} = \Psi_{kk}^{\omega_k} \neq \Psi_{kk}^{\omega'_k} = \Psi_k^{\omega'_k *}, \ \ \forall \omega_k, \omega'_k \in \Omega_k. \tag{44}$$

However, for household k (living in city k in equilibrium), either $\tilde{\sigma}_k \subseteq \mathcal{F}_j$ or $\tilde{\sigma}_k \nsubseteq \mathcal{F}_j$ is ture. When $\tilde{\sigma}_k \subseteq \mathcal{F}_j$, (44) contradicts with the condition of local information. When $\tilde{\sigma}_k \nsubseteq \mathcal{F}_j$, $\exists \omega_k, \omega'_k \in \Omega_k$ such that $E[u_{jk}|\mathcal{F}_j]$ is the same for these two states. From (41) and footnote 13, it is shown that $\exists \omega_k, \omega'_k \in \Omega_k$, $\Psi_{kk}^{\omega_k} = \Psi_{kk}^{\omega'_k}$, a contradiction with (44). In fact, these non-fully revealing equilibrium prices reveal completely nothing in equilibrium.

From (44), for any pair $\omega_k, \omega'_k \in \Omega_k$, without loss of generality, we can have $\Psi_{kk}^{\omega_k} > \Psi_{kk}^{\omega'_k}$. Then there exists $\epsilon \equiv \min\{(|\Psi_{kk}^{\omega_k} - \Psi_{kk}^{\omega'_k}|)_{\omega_k, \omega'_k \in \Omega_k}\}$ such that for all economies under local perturbations less than ϵ, there exists a unique non-fully revealing rational expectations equilibrium. *Q.E.D.*

Local information is sufficient but not necessary for the existence of a non-fully revealing rational expectations equilibrium. An example is shown in Figure 6, where household 1 (2) has full information about city y (x) and in city x (y) a quasi-linear utility in the composite goods. On the other hand, a necessary and sufficient condition for the states in city k being revealed is that for household j such that $\tilde{\sigma}_k \subseteq \mathcal{F}_j$, (i) the realized rate of marginal substitution are not the same, $\forall \omega \in \Omega$; (ii) $j = k$, which implies that in equilibrium, by Assumption 1, household j lives in city k.

Lemma 2 *Given Assumption 1 and consider local perturbations, when $\tilde{\sigma}_k \subseteq \mathcal{F}_k$ and the condition of local information is violated for household k, then there exists an open subset of economies that in any rational expectations equilibrium, at least the state in city k is revealed.*

Proof. When local information condition is violated, since $\tilde{\sigma}_k \subseteq \mathcal{F}_j$, the realized marginal rates of substitution are different $\forall \omega_k \in \Omega_k$. From Assumption

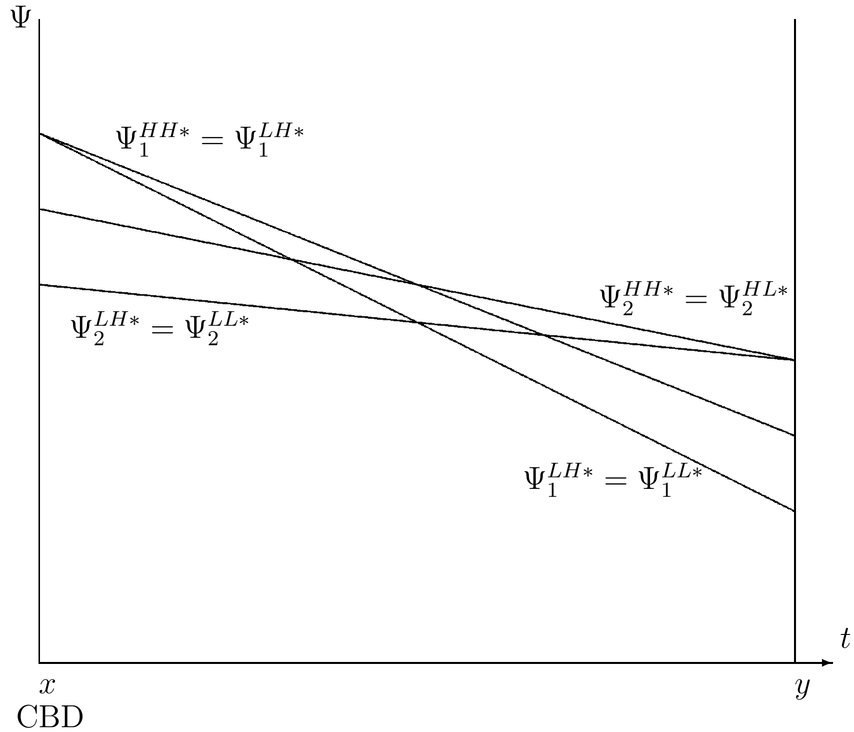

Figure 6: When every information is non-local, we can still have a non-fully revealing rational expectations equilibrium.

1 and Lemma 1, it is shown that in the rational expectations equilibrium, the yielded bid rents are different for all states. Consider an open subset of local perturbations such that household k lives in city k in equilibrium, it is proved that in any equilibrium, the state in city k is revealed for sure. *Q.E.D.*

Theorem 3 *Given Assumption 1 and consider local perturbations, when the condition of local information is violated for all households, given $\tilde{\sigma}_k \subseteq \mathcal{F}_k$, $\forall k$, there exists an open subset of economies that possess a unique fully revealing rational expectations equilibrium; On the other hand, if $\exists k$ such that $\tilde{\sigma}_k \nsubseteq \mathcal{F}_k$, then there exists an open subset of economies that possess a non-fully revealing rational expectations equilibrium.*

This theorem is directly implied by Lemma 2 for all households. When the condition of local information is violated, households' realized marginal rates of substitution are precise enough in distinguishing different states; furthermore, ordered steepness of bid rents ensures the existence of a location equilibrium and $\tilde{\sigma}_k \subseteq \mathcal{F}_k$, $\forall k$, ensures that in equilibrium the informed households are exactly living in the cities that they have information about. Therefore, under local perturbations and centered at Example 2 where households' information σ-algebra are switched, it can be checked that there exists an open subset of economies possessing fully revealing rational expectations equilibrium. On the other hand, similar to the proof of Theorem 1, when there exists a household such that $\tilde{\sigma}_k \nsubseteq \mathcal{F}_k$, from Lemma 2, there exists an open subset of economies that possess a at best partially revealing rational expectations equilibrium where the state in city k cannot be revealed in equilibrium.

As discussed in the beginning of this paper, in most cases in reality, the households in city k have information that others don't have. This means that though $\tilde{\sigma}_k \subseteq \mathcal{F}_k$, $\forall k$, is tight in analyses, it is a more realistic condition, especially when a long-run equilibrium is concerned. Moreover, since an open subset of fully revealing and an open subset of non-fully revealing economies are found in Theorem 3, it is shown that in fact, both the sets of fully revealing and non-fully revealing economies cannot be dense under the structure of urban economics.

Finally, it can be noticed that all Theorem 1, 2, and 3 are under the consideration of local perturbations and emphasizing the existence of an open subset of non-fully revealing rational expectations equilibrium economies, for non-fully revealing equilibrium is more interesting in highlighting the positive value and the strategic behavior of information. On the other hand, when non-local perturbations are considered, though they are not so reasonable in urban economics, the results are immediately the same as the ones in standard general equilibrium models. That is, there is an open and dense subset of economies that possess a fully revealing rational expectations equilibrium. All the results can be summarized in Table 1: Though local perturbations ensure the existence of a non-fully revealing rational expectations equilibrium, the condition of local information is needed to ensure that the unique rational expectations equilibrium is non-fully revealing.

From above, since these "local properties" are the sources for the generic existence of an open set of non-fully revealing rational expectations equilibrium, it is concluded that location, together with local properties, can play a role in distorting the efficiency of market prices in transmitting information from informed to uninformed households. On the other hand, in economic circumstances where there is no location structure and no local property, the efficiency of prices in information transmission is generically maximized in equilibrium.

	Local perturbations	Non-local perturbations
Local information	An open subset of non-fully revealing REE	An open and dense subset of fully revealing REE (Standard model)
Non-local information	Open subsets of fully revealing REE and non-fully revealing REE	An open and dense subset of fully revealing REE (Standard model)

Table 1: Summary of the types of rational expectations equilibria for different perturbations and different information conditions.

5 Conclusions

Allen (1981) proves the existence of an open and dense subset of standard economies that possess revealing rational expectations equilibria. Since an open subset of economies without fully-revealing rational expectations equilibrium is found in Theorem 1, this paper shows that Allen's theorem about the existence of a dense subset of economies possessing fully-revealing rational expectations equilibrium does not extend to urban economics. Furthermore, since an open subset of economies with fully revealing rational expectations equilibria can easily be constructed, we cannot challenge the existence of an open subset of economies that possess fully-revealing rational expectations equilibria in the context of urban economics. Therefore, both the sets of fully revealing and non-fully revealing economies cannot be dense under the structure of urban economics.

Furthermore, this paper highlights the important "local properties" for the existence of rational expectations equilibria in urban economics. Under generically local perturbations, the existence of a unique rational expectations equilibrium is proved with the assumption of ordered relative steepness and the local information condition. Either the rational expectations equilibrium is fully revealing or non-fully revealing depends on where the informed households live. When the city that their information is relevant with matches with the city where they live, the rational expectations equilibrium is fully revealing; Otherwise, the rational expectations equilibrium is non-fully revealing. In summary, location and local properties can play a role in distorting the efficiency of market prices in transmitting information from informed to uninformed households.

One potential extension of this paper is to replace households to be firms, who can own fractions of the stocks of other firms. In this case, even when firms are not located in the cities they are informed about, similar to Berliant and De (1998), their demand for other firms' stocks and the equilibrium stock prices will reveal their private information. Another extension is to consider a continuum of households, however, the strong intuition that the mismatching of local-informed households and the corresponding locations yields an open subset of economies possessing non-fully revealing rational expectations equilibrium.

Appendix A. Proof of Lemma 1

Comparing Definition 1 and Definition 2, since condition (iii) is the same as equations (3) and (4), we need to prove that conditions (i) and (ii) are equivalent to equations (1) and (2).

First, to prove this, given that (1) and (2) are satisfied but either (i) or (ii) are not true, we want to show contradictions. If (i) is not true, there exists $\Omega_0 \subseteq \Omega$ with $\mu(\Omega_0) > 0$ such that $\psi_k^*(\omega) \cdot \varphi_{jk}^*(\omega) > Y - T_k$, $\forall \omega \in \Omega_0$. Then for these $\omega \in \Omega_0$, we can have $\Psi_k^{\omega *} s_{jk}^{\omega *} + z_{jk}^{\omega *} > Y - T_k$, which implies

$$\Psi_k^{\omega *} > \frac{Y - T_k - z_{jk}^{\omega *}}{s_{jk}^{\omega *}}, \quad \forall \omega \in \Omega_0,$$

a contradiction with (1), chosen a utility level the same as the optimalized level by Definition 1 ($u = u^*$).

On the other hand, if (ii) is not true, then $\exists j \in \{1, 2\}$ and $\exists \varphi_j'(\omega)$ within the budget constraint such that

$$\int_\Omega u_j^\omega(\varphi_j')\, d\mu(\omega|\mathcal{F}_j \vee \sigma(\Psi^*)) > \int_\Omega u_j^\omega(\varphi_j^*)\, d\mu(\omega|\mathcal{F}_j \vee \sigma(\Psi^*)). \tag{45}$$

For this household j and for city k where he/she lives, we can choose $u = u^*$, and then by strict concavity and strict monotonicity, there exists $\epsilon > 0$ and $\varphi_j''(\omega) \equiv \frac{\varphi_j'(\omega)+\varphi_j^*(\omega)}{2} - \epsilon$ such that $E[u(\varphi_j''|\mathcal{F}_j)] = u^*$. Since $\psi_k^*(\omega)\varphi_{jk}''(\omega) < Y - T_k$ implies $\Psi_k^{\omega *} < \frac{Y-T_k-z_{jk}^{\omega''}}{s_{jk}^{\omega''}}$, let $\Psi_k^{\omega''} \equiv \frac{Y-T_k-z_{jk}^{\omega''}}{s_{jk}^{\omega''}}$, we find $\Psi_k^{\omega''} > \Psi_k^{\omega *}$ for a given $u = u^*$. It is proved that $\varphi_{jk}^{\omega *}$ does not maximize $\Psi_{jk}^{\omega *}$, a contradiction with equation (2).

Secondly, given (i) and (ii) are true, but either (1) or (2), is not satisfied, we want to prove that there exists a contradiction. If (1) does not hold, there exists k and j such that $\Psi_{jk}^\omega > \Psi_k^{\omega *}$ but j does not live city k. Then for this household j, since he/she can pay less for the housing in k than the price that makes he/she indifferent between the housing in two cities, household j has an incentive to move into city k, a contradiction with condition (ii) that φ_j^* maximizes j's conditional expected utility.

If (2) does not hold, since the budget line with $\Psi_{jk}^{\omega*}$ is not tangent with the difference curve for a given u, by strict concavity, there exists $\varphi_{jk}^{\omega'} \neq \varphi_{jk}^{\omega*}$ such that $E[u(\varphi_j'|\mathcal{F}_j)] = u^*$. By strict concavity again, choose $\varphi_{jk}^{\omega''} \equiv \frac{\varphi_{jk}^{\omega*}+\varphi_{jk}^{\omega'}}{2}$, then $\varphi_{jk}^{\omega''}$ is available for household j to achieve $E[u(\varphi_j''|\mathcal{F}_j)] > u^*$, a contradiction with (ii) that φ_j^* maximizes household j's expected utility.

References

[1] Allen, B. (1981): "Generic Existence of Completely Revealing Equilibria for Economies with Uncertainty when Prices Convey Information," *Econometrica*, 49, 1173-1199.

[2] Allen, B., AND J. S. Jordan (1998): "The Existence of Rational Expectations Equilibrium: A retrospective," in *Organizations with Incomplete Information*, ed. by M. Majumdar. Cambridge: Cambridge University Press, 42-60.

[3] Alonso, W. (1964): *Location and Land Use*. Cambridge: Harvard University Press.

[4] Aumann, R. J. (1976): "Agreeing to Disagree," *The Annals of Statistics*, 4, 1236-1239.

[5] Berliant, M., AND S. De (1998): "On the Revelation of Private Information in Stock Market Economies," *Journal of Mathematical Economics*, 30, 241-256.

[6] Berliant, M., AND M. Fujita (1992): "Alonso's Discrete Population Model of Land Use: Efficient Allocations and Competitive Equilibria," *International Economic Review*, 33, 535-566.

[7] Blomquist, G. C., M. C. Berger, AND J. P. Hoehn (1988): “New Estimates of Quality of Life in Metropolitan Areas,” *American Economic Review*, 78, 89-107.

[8] Cronon, W. (1991): *Nature’s Metropolis: Chicago and the Great West*. New York: W. W. Norton.

[9] Debreu, G. (1972): “Smooth Preferences,” *Econometrica*, 40, 603-615.

[10] Fujita, M. (1985): “Existence and Uniqueness of Equilibrium and Optimal Land Use,” *Regional Science and Urban Economics*, 15, 295-324.

[11] Fujita, M., AND T. E. Smith (1987): “Existence of Continuous Residential Land-Use Equilibria,” *Regional Science and Urban Economics*, 17, 549-594.

[12] Fujita, M. (1989): *Urban Economic Theory*. Cambridge: Cambridge University Press.

[13] Fujita, M., P. Krugman, AND A. J. Venables (2001): *The Spatial Economy: Cities, Regions, and International Trade*. Cambridge: The MIT Press.

[14] Jordan, J. S. (1980): “On the Predictability of Economic Events,” *Econometrica*, 48, 955-972.

[15] Radner, R. (1979): “Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices,” *Econometrica*, 47, 655-678.

[16] Roback, J. (1982): “Wages, Rents, and the Quality of Life,” *Journal of Political Economy*, 90, 257-278.

[17] Rosen, S. (1979): "Wage-Based Indexes of Urban Quality of Life," in *Current Issues in Urban Economics*, ed. by P. Mieszkowski and M. Straszheim, Baltimore: Johns Hopkins University Press.