

Excess Demand and Rationing in Equilibrium in the Market for Public Housing*

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Abstract

We develop a model for the market of public housing that captures excess demand for public housing and rationing in equilibrium. We characterize the equilibrium and show that a unique equilibrium exists if the housing authority follows an equal treatment policy and does not discriminate based on current residence. We then develop a maximum likelihood estimator for the parameters of the model. The estimator accounts for enriched sampling and imposes all equilibrium conditions in estimation. We estimate the parameters of the model based on a unique restricted use panel data set of low-income households in Pittsburgh. We find that for each household that leaves public housing there are on average 14 households that would like to move into the vacated unit. Demolitions of existing units increase the degree of rationing and are likely to result in welfare losses.

JEL classification: C33, C83, D45, D58, H72, R31

1 Introduction

The U.S. Department of Housing and Urban Development (HUD) funds the construction and maintenance of affordable public housing communities in cities and metropolitan areas.¹ Low income households are eligible for public housing assistance if their income is below a threshold that depends on family status, number of children, and region. Given the current standards for determining eligibility, there is typically a large number of eligible households in each metro area. Supply of public housing units is primarily determined by the current and past political decisions that have allocated funding for HUD and the local housing authorities. Since rents in public housing are typically a fixed percentage of household income, there is no price mechanism which guarantees that public housing markets clear. As the consequence the demand for public housing often exceeds supply. Hence, there is rationing in equilibrium in many local markets.

The first objective of this analysis is to develop an equilibrium model that is consistent with rationing and excess demand for public housing by low income households. To incorporate supply restrictions we model the administrative behavior of the local housing authority. A household can move into public housing if and only if the housing authority offers the household a vacant apartment. The ability of the housing authority to offer apartments to eligible households is largely determined by voluntary exit decisions of households that currently live in housing communities. Exit from public housing is a stochastic event since it is partially determined by idiosyncratic preference and income shocks that are not observed by the administrators. The housing authority's objective is to fill the vacant units. If the potential demand exceeds the available units at each point of time, the housing authority has to ration access to public housing.

We formalize these restrictions by assuming that eligible households receive stochastic offers to move into public housing. If total supply of public housing is fixed and vacancy rates are constant over time, the housing authority adjusts the offer probabilities in equilibrium so

¹This paper focuses the market for public housing communities. The other main rental assistance program funded by HUD provides vouchers for household to rent in the private market.

that the inflow into public housing equals the voluntary outflow. We define an equilibrium for our model and characterize its properties. We show that there exists a unique equilibrium if the housing authority follows an equal treatment policy and does not discriminate based on current residence.

The second objective of the paper is to show how to identify and estimate the parameters of the model.² Identification is not straight forward since we do not observe which households received offers to move into housing communities. We only observe those offers that were accepted and resulted into a move. This type of selection problem is also encountered in labor search and occupational choice models. However, there is an important difference between labor search and housing search models. Labor economists observe accepted wages which are treated as measures of job quality.³ As a consequence they can rely on an “identification at infinity” argument since selection does not arise in the limiting case.⁴ In contrast, urban economists have only access to imperfect measures of housing quality and some important components of housing quality are not observed by the econometrician. As a consequence, we need to develop a new identification strategy. The basic insight of our approach is that offer probabilities are endogenous and are constrained to satisfy equilibrium conditions. Hence, offer probabilities are not exogenously given parameters, but can be expressed as functions of the structural parameters of the search models. Imposing these equilibrium conditions then establishes identification for our model. We can, therefore, estimate the parameters of the model using a maximum likelihood estimator.

The final objective of the paper is to quantify the importance of supply side restrictions. We have obtained a unique data set from the Housing Authority of the City of Pittsburgh (HACP).⁵ We supplement these data with a sample of eligible low income households in

²Since there is rationing in equilibrium, standard discrete choice estimators proposed by McFadden (1973, 1974, 1981) and Berry, Levinsohn, and Pakes (1995) do not directly apply, and need to be modified.

³For a discussion of identification and estimation of labor search model see, among others, Eckstein and Wolpin (1990) and Postel-Vinay and Robin (2002)

⁴See, for example, Heckman and Honore (1990) for a discussion of identification in the Roy model.

⁵Olsen, Davis, and Carrillo (2005) use restricted use data from HUD to study the impact of variations in local housing policies on household behavior.

the SIPP since we also need to observe eligible households outside of public housing. Since we rely on two separate data sets to estimate the model, we face two additional problems. First, the HACP sample is a choice based sample and estimation needs to account for this sampling scheme. Second, the econometrician does not observe the full choice set in each sample. However, there is sufficient overlap in the choice sets such that the model can be estimated using an enriched sampling maximum likelihood estimator proposed by Cosslett (1981).⁶

We find that our approach yields plausible estimates for the structural parameters of interest. Households headed by single mothers or blacks have strong preferences for public housing. The income coefficient shows that there are strong incentives for households to leave public housing as their income grows larger. These incentives are off-set by the presence of significant moving costs that constrain potential relocations of households. Our estimates imply that there are serious supply restrictions which prevent the mobility of those willing to transfer or move into public housing. We find that for each household that leaves public housing there are on average 14 households that would like to move into the vacated unit. Hence, there is a significant excess demand in equilibrium.

Since our model fits the data reasonably well and yields plausible parameter estimates, we can use the estimates to conduct some counter-factual policy experiments. We consider the impact of demolishing old, undesirable public housing communities. We find that demolitions increase the excess demand for public housing and thus yields to more rationing in equilibrium. Moreover, displaced black females are disproportionately disadvantaged, and the resulting equilibrium demographic distribution in the remaining public housing communities leads to an increase in female, minority, parent, and low-income residents.

The remainder of the paper is organized as follows. Section 2 discusses our main data sources. Section 3 provides an equilibrium model that treats public housing as a differentiated product that is subject to rationing. Section 4 discusses identification and derives the ML estimator for this model. The empirical results are presented in Section 5. Section

⁶Since our estimator is not straight forward to implement, we have conducted a small Monte Carlo study which shows that our estimation approach is feasible given the specifics of our sampling design.

6 quantifies the excess demand for public housing in equilibrium. We discuss the welfare implications of demolitions in Section 7. We offers some conclusions that can be drawn from the analysis in Section 8.

2 Data

The U.S. Housing Act of 1937 formed the U.S. Public Housing Program that funds local governments in their ownership and management of buildings to house low-income residents at subsidized rents.⁷ Currently, the U.S. Department of Housing and Urban Development funds the efforts of hundreds of city and county Housing Authorities in the United States. In Pennsylvania alone, there are 92 distinct Housing Authorities. In 2006, the estimated HUD budget for public housing was \$24.604 billion.⁸ Within the public housing program, this funding supports administration, building maintenance, and even law enforcement.

The empirical analysis presented in this paper focuses on communities owned and managed by the Housing Authority of the City of Pittsburgh.⁹ In 2005 HUD provided the HACP with \$83.7 million in grants for public housing, housing vouchers, and other programs. In the same year, HACP received \$8.3 million from tenant payments. About 20,000 city residents are housed by HACP programs, about 6.3% of the total city population. The main programs offered by the HACP are public housing as well as the Housing Choice Voucher Program, formerly and still popularly referred to as the Section 8 Housing Vouchers. Currently about half of the population served by the HACP lives in public housing which are the focus of this paper.

The public housing stock in the City of Pittsburgh is heterogeneous, including small houses converted into several apartment units, large high-rises, and large communities of low-rise housing spread continuously over several blocks. Only a small number public hous-

⁷Olsen (2001) provides a detailed description of the history and current practices of the various different U.S. Public Housing Programs.

⁸HUD (2007) provides details. Note that this figure does not include housing voucher programs, low-income community development programs, or other none-state owned and managed housing programs.

⁹We use the term "community" to denote a public housing complex.

ing communities were demolished during the course of our survey.¹⁰ As a consequence the supply of public housing has been approximately fixed during our study period. There is a great variety of communities ranging in size from four units to over 600 units in various neighborhoods across the city. These communities are usually designated as either 'family' communities or 'senior' communities, where senior communities target households age 62 or older. There are 34 separate sites. 19 of these sites are family units, 11 are designated for seniors and 4 of them are mixed. There are 16 large communities with more than 100 units, 8 are medium sized, and 10 are small with less than 40 units.

Heterogeneity in public housing also arises due to differences in local amenities. The 34 public housing communities in the HACP are located across 19 of Pittsburgh's 32 wards and across 28 census tracts. We construct amenity measures for each housing community. Crime is measured by the level of violent or property crime per year per 1,000 people.¹¹ School quality is given by the average of public school test scores, weighted by enrollment in different school grades for high schools. We also study the ratio of the average home sales price to the city-average home sales price.¹²

Table 1 illustrates that the public housing communities in Pittsburgh vary widely in terms of neighborhood amenities such as crime, school quality, property values and demographic characteristics.¹³ Table 1 also reports characteristics of census tracts in which the

¹⁰Much of the demolition was motivated by the insight that growing up in public housing might be negative for children, although this conjecture is controversial in the literature (Currie and Yelowitz, 2000). For an analysis of the the impact of public housing demolitions in Chicago see Jacob (2004).

¹¹The crime indices used here are taken from Epple, Peress, and Sieg (2009a) who construct a crime index for every municipality in Allegheny County and for every ward in the City of Pittsburgh. The index is constructed based on crime data from the years 1999, 2000, and 2001; the index is adjusted to the 2001 - level and indicates crimes per year per 1,000 people. There is a violent crime index that includes murder, rape, robbery, and aggravated assault; there is a property crime index that includes burglary, larceny, theft, and motor-vehicle theft into Property Crimes.

¹²The ratios of wards' median home prices are calculated using home sales prices recorded by the Allegheny County Office of Property Assessments.

¹³There is much evidence that suggests that households make residential decisions based on neighborhood characteristics and local public goods. This evidence is based on estimated locational equilibrium models such as Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Wu and Cho (2003), Sieg, Smith, Banzhaf,

Table 1: Neighborhood Amenities Across Public Housing Communities

	Mean	Std	Min	Max
High School	1.22	0.06	1.12	1.32
Violent Crime	969.02	525.88	248.71	1953.69
Property Crime	4251.03	1456.33	2499.17	6962.65
All Crime	1466.68	613.27	604.74	2539.56
Ratio of Sales Price	1.14	0.49	0.45	2.21
Median Income	19218	9855	7042	38388
Percent White	0.41	0.36	0.01	0.97
Med age	35.23	10.38	14.10	51.90
Percent Own Home	0.34	0.25	0.00	0.79
Percent Homes Vacant	0.20	0.17	0.04	0.75

the housing communities are located. At the Census tract level, we examine median income, percent minority, building vacancy, percent of households that own their dwelling, and median age. While these measures also reflect the residency of public housing households in the selected census tracts, the percent of residents who are in public housing is sufficiently small to render these statistics still interesting.

The HACP data contain records of household entry, exits, and transfers from June 2001 to June 2006 within the 34 HACP public housing communities actively used during this time period. The data set also includes annual updates of each of these households as well as any non-periodic reports that update information about household composition or income that is reported to the HACP. These records contain most of the information and Walsh (2004), Calabrese, Epple, Romer, and Sieg (2006), Ferreyra (2007), Walsh (2007), and Epple, Peress, and Sieg (2009a). Bergstrom, Rubinfeld, and Shapiro (1982), Rubinfeld, Shapiro, and Roberts (1987), Nesheim (2001), Bajari and Kahn (2004), Bayer, McMillan, and Reuben (2004), Schmidheiny (2006), Bayer, Ferreira, and McMillan (2007a), and Ferreira (2009) are examples of related empirical approaches which are based on more traditional discrete choice models or hedonic frameworks.

fields requested of all U.S. Housing Authorities including age, race, disability, household composition including age and relationship of family members and housemates, earnings, and income adjustment exclusions including disability, medical, and childcare expenses. We also observe the monthly rent being charged to a particular household, the number of bedrooms of the housing unit, whether the community is targeted to seniors, whether it is handicap-accessible, and the address and unit number. There are 7,070 households observed at least once during this time period; there are 2907 households that move in for the first time, 3155 households that move out, and 1244 that transfer from one public housing unit to another.

Table 2: Descriptive Statistics of HACP Demographics

	All	Family	Mixed	Senior
Age	48.86 (20.76)	40.42 (16.98)	49.06 (20.53)	71.15 (11.77)
Percent Female	80.59 %	84.87 %	83.85 %	64.90 %
Percent Married	2.66 %	2.20 %	2.65%	3.93%
Number of Adults	1.16 (0.44)	1.17 (0.45)	1.21 (0.50)	1.06 (0.23)
Number of Children	0.95 (1.36)	1.00 (1.22)	1.59 (1.71)	0.00 (0.00)
Percent With Children	43.95%	53.46%	58.31%	0.00 %
Percent Black	88.53%	96.67%	97.00%	55.59%
Annual Income	9082 (7776)	8516 (8957)	9714 (6968)	9784 (4602)
Disabled	20.12%	18.10%	16.06%	30.74%

Standard deviations are given in parenthesis.

Table 2 summarizes key descriptive statistics for the full sample and for three subsamples that are differentiated by community type. Although some families live in senior housing and some seniors live in non-senior housing, age and family composition distributions are bimodal with respect to these two types of communities. In mixed communities, demographic variables look similar to a weighted average of senior and family communities, however there are more cohabiting adults and a higher number of children in mixed

housing than in family-only or senior-only housing. The mean age in senior housing is 31 years greater than the mean age in non-senior housing. The majority of households in both senior-only and family-only communities are female, but females are a much larger majority in family-only communities. Blacks households are a very high proportion of residents in family and mixed housing, while senior units have nearly equal proportions of black and white households. Marriage rates are low, 2.20% in family housing and 3.93% in senior housing; there are more cohabiting adults in family housing than in senior housing.¹⁴ There are fewer households in non-senior housing that have children than one might expect (about 53%).¹⁵

From the HACP, we only observe households that have lived in public housing at some point during the sample period. Once households leave the housing communities, the HACP does not conduct any follow-up surveys. To learn about households that are eligible for public housing, but do not live in one of the housing communities, we turn to the 2001 Survey of Income and Program Participation (SIPP). The SIPP is a survey managed by the U.S. Census Bureau that interviews households every four months for 3 years. Each month, households are asked about their previous four months' family composition, sources of income, and participation in government programs such as public housing and school lunch programs. We create a sample based on the SIPP that contains all households that eligible for housing aid.

The SIPP contains only 14 households that participate in public housing in Pittsburgh at some point during the sample period. There are 156 Pittsburgh households eligible for public housing in the first quarter.¹⁶ We, therefore, constructed a sample that also includes

¹⁴There is a strong incentive for families to not report the existence of a cohabiting adult or partner, as it would lead to an increase in rent if the cohabiting adult earns an income. As a result, the number of cohabiting adults as well as household income are surely larger than our estimates from the data.

¹⁵Our sample differs from other studies in that Pittsburgh public housing seems to house a higher percent of black households, female-headed households and households with children; but a much lower percent of married households. For example, Hungerford '96's sample from the 1986-1988 SIPP panel was 52% female, 23% black, 32% married and the mean number of children was 0.21 (Hungerford, 1996).

¹⁶There is some attrition in the SIPP. In the last quarter there are only 125 Pittsburgh households eligible for public housing.

households from metropolitan areas with similar characteristics.¹⁷ Table 3 provides some descriptive statistics for our SIPP used in this analysis and compares the SIPP sample to Census and HACP data.

Table 3: Descriptive Statistics SIPP Compared to Census and HACP

	Census All	SIPP All	SIPP Private	SIPP Public	HACP Public
Age	50.83	52.70	52.72	52.19	48.86
Percent Female	54.6%	59.94%	59.06%	76.56%	80.59%
Percent Married	22.6%	30.79%	32.09%	6.25%	2.66%
Number of Adults	1.450	1.274	1.284	1.094	1.160
Number of Children	0.495	0.617	0.616	0.641	0.950
Percent With Children	24.73%	30.32%	30.27%	31.25%	43.95%
Percent Black	32.64%	28.28%	27.05%	51.56%	88.53%
Annual Income	14079	18979	19391	11184	9082

We find that low-income households that rent in the private market are on average more likely to be married, are less likely to be black, and have substantially higher income than households in public housing. Comparing the SIPP with the HACP sample we find that the SIPP sample is slightly older and as a consequence average income is slightly higher and children are fewer than in the HACP. Comparing the SIPP with the Census, the SIPP contains slightly older heads of household, more female heads of household, more married householders, households with more children, and fewer black households. However, the differences between the SIPP sample and the Census sample of eligible households in Pittsburgh are relatively small.

To estimate our model we aggregate the 34 communities into six community types: family large (PH 1), family medium (PH 2), family small (PH 3), mixed (PH 4), senior

¹⁷Details are reported in Appendix B.

Table 4: Transition Matrix

	Private	PH 1	PH 2	PH 3	PH 4	PH 5	PH 6
Private	0	677	144	24	300	59	191
PH 1	855	16264	16	2	75	7	10
PH 2	233	16	5371	3	17	8	7
PH 3	44	2	29	1438	1	0	2
PH 4	572	16	8	1	12156	5	9
PH 5	105	1	0	0	1	2017	29
PH 6	302	0	0	1	47	37	8129

large (PH 5), and senior small (PH 6).¹⁸ Our aggregation is based on a classification used by the HACP. Large, medium, and small low-rise non-senior communities (PH 1-4) primarily house families with children. However, they also include a significant percent of households without children (36% to 42%). Although the demographics of senior (PH 5-6) and family (PH 1-4) housing differ, there is some overlap. Most senior-dominated (PH 5-6) communities include a significant percentage of non-senior adults without kids (13% to 37%); also most family-only communities include some senior households (ranging from 0 - 20%), about a third of which are caring for children.

Table 4 shows the transition matrix for the HACP data. We find that locational choices are persistent since most households stay with their past choices. However, the off-diagonal elements of the transition matrix indicate that there is a fair amount of entry into and exit from public housing. Moreover, there are a number of transitions within public housing communities. These transfer are largely voluntary and indicate that household seem to differentiate among the different community types. This is consistent with of our previous findings that housing communities should be treated as differentiated products. We also consider mobility patterns in the SIPP sample. Here we observe 89 transitions from private

¹⁸The main reason for aggregation is that some of the communities are too small to accurately estimate the community-specific fixed effects.

to public housing and 98 transitions from public to private housing.

3 An Equilibrium Model with Rationing

3.1 Household Heterogeneity and Preferences

We consider a model with a continuum of low-income households. Each household is eligible for housing aid and can thus, in principle, live in one of the public housing communities or rent an apartment in the private market. Denote the outside private market option with 0. Let J be the number of different housing communities that are available in the public housing program. Let $d_{jt} \in \{0, 1\}$ denote an indicator variable which equals one if the household chooses alternative j at time t and is zero otherwise. Let the vector $d_t = (d_{0t}, \dots, d_{Jt})$ characterize choices of an household at t . Since the alternatives are mutually exclusive,

$$\sum_{j=0}^J d_{jt} = 1 \quad (1)$$

Households differ along a number of characteristics x_t such as income, age, number of kids, number of adults, gender of household head, marital status, and race. Moreover, household preferences are subject to idiosyncratic shocks denoted by ϵ_t . Households face relocation costs if they decide to move. Thus lagged choices, denoted by d_{t-1} , are relevant state variables.

Households have preferences defined over all potential elements in the choice set. We model household preferences using a standard random utility specification.

Assumption 1 *Let $u(d_t, x_t, d_{t-1}, \epsilon_t)$ denote the household utility function. We assume that the utility function is additively separable in observed and unobserved states and thus allows the following representation:*

$$u(d_t, x_t, d_{t-1}, \epsilon_t) = \sum_{j=0}^J d_{jt} [u_j(x_t, d_{t-1}) + \epsilon_{jt}] \quad (2)$$

This specification implicitly treats public housing as a differentiated product which captures the fact that housing differs along dimensions such as physical attributes of housing units in a community and neighborhood specific amenities such as crime and school quality.¹⁹

3.2 The Housing Authority

Next we model the behavior of the local housing authority. A key feature of our model is that all potential choices may not be available to the households at any given point of time. A household that is currently renting in the private market may not have access to public housing even if the household meets all eligibility criteria.²⁰ We, therefore, need to formalize the fact that access to public housing is restricted by a local housing authority. To model the behavior of this authority we follow the labor literature on search and assume that offers arrive to households according to a well defined stochastic process.

Assumption 2 *There is a constant probability, Π_{0jt} , of getting an offer in housing community j at time t while renting in the private market. Each household can get at most one offer at each point of time:*

$$\sum_{j=1}^J \Pi_{0jt} \leq 1 \quad (3)$$

Let $\Pi_{00t} = 1 - \sum_{j=1}^J \Pi_{0jt}$ denotes the probability of not getting an offer at all.

Similarly, a household that is currently living in a public housing community may not always have the opportunity to move to another community.

Assumption 3 *The probability of obtaining an offer from housing community j while living*

¹⁹There is much evidence that neighborhood characteristics are important in determining locational choices. See, among others, Epple and Sieg (1999), Epple et al. (2001), Bayer et al. (2004), Bajari and Kahn (2004), Ferreyra (2007), and Bayer et al. (2007a).

²⁰In practice, all eligible households are typically assigned to a waiting list. A household will only receive an offer to move into public housing if it is on top of the waiting list.

in public housing i is given by Π_{ijt} . Households get at most one offer at each point of time:

$$\sum_{j=1}^J \Pi_{ijt} \leq 1 \quad (4)$$

Note that household can always leave public housing, i.e. $\Pi_{j0t} = 1$ for $j = 1, \dots, J$.

3.3 Entry, Transfers, and Exit

Given the assumptions above, the probability that a households in community i prefers j at time t is:

$$Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) = Pr(u_j(x_t, d_{t-1}) + \epsilon_{jt} \geq \max [u_i(x_t, d_{t-1}) + \epsilon_{it}, u_0(x_t, d_{t-1}) + \epsilon_{0t}]) \quad (5)$$

The potential demand is then characterized by the fraction of households living in community i that prefer j at time t is then:

$$F_{ijt} = \int Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) f(x_t | d_{it-1} = 1) dx_t \quad (6)$$

where $f(x_t | d_{jt-1} = 1)$ denotes the conditional density function of x_t of households with characteristics x_t that live in j at the beginning of the period. Similarly, the fraction of households that rent in the private market and that want to move to community j at time t is given by the following expression:

$$F_{0jt} = \int Pr(u_0(x_t, d_{t-1}) + \epsilon_{0t} \leq u_j(x_t, d_{t-1}) + \epsilon_{jt}) f(x_t | d_{0t-1} = 1) dx_t \quad (7)$$

Let P_{it} denote the fraction of households living in i at the beginning of period t . A key feature of public housing is that not all households that want to move from i to j have the opportunity to move due to rationing. Recall that the probability of being offered a spot in j while living in i is denoted by Π_{ijt} . The total realized demand (or inflow) from community i to community j at time t is therefore $P_{it} \Pi_{ijt} F_{ijt}$. Summing over all current housing choices other than j gives the total inflow into housing community j :

$$IF_{jt} = \sum_{i=0, i \neq j}^J P_{it} \Pi_{ijt} F_{ijt} \quad (8)$$

Similarly we can characterize the total voluntary outflow from community j which is given by:

$$OF_{jt} = P_{jt} [OF_{j0t} + \sum_{i=1, i \neq j}^J \Pi_{jit} F_{jit}] \quad (9)$$

where the outflow to the private sector, OF_{j0t} , is defined as:

$$OF_{j0t} = \Pi_{j0t} \int Pr(u_0(x_t, d_{t-1}) + \epsilon_{0t} \geq u_j(x_t, d_{t-1}) + \epsilon_{jt}) f(x_t | d_{j,t-1} = 1) dx_t \quad (10)$$

$$+ \sum_{k=1, k \neq j}^K \Pi_{jkt} \int Pr(u_0(x_t, d_{t-1}) + \epsilon_{0t} \geq \max [u_j(x_t, d_{t-1}) + \epsilon_{jt}, u_k(x_t, d_{t-1}) + \epsilon_{kt}]) f(x_t | d_{j,t-1} = 1) dx_t$$

To close the model, we need to impose an assumption on the supply of public housing and the vacancy rates.

Assumption 4 *The supply of public housing is constant and the number of vacancies is constant in each housing community at each point of time.*

This assumption implies that outflows must equal inflows in equilibrium.²¹ We can relax this assumption and allow for exogenous changes in inflows or outflows due to demolitions or building expansions.²²

3.4 Definition of Equilibrium

An equilibrium for this model can, therefore, be defined as follows:

Definition 1 *Given an initial distribution of household types, an equilibrium for this model consists of a vector of offer probabilities such that*

- *Households choose the preferred housing option among the set of available options.*

²¹The assumption of a constant housing stock is common in many theoretical papers that study housing market equilibrium in urban metropolitan areas. See, for example, Nechyba (1997a, 1997b), Nechyba (2003), Bayer and Timmins (2005), and Ferreyra (2007).

²²We provide an application in Section 7 that studies the impact of demolitions.

- For each housing community j , the housing authority determines offer probabilities such that inflows of households equal the outflows of households:

$$\sum_{i=0, i \neq j}^J P_{it} \Pi_{ijt} F_{ijt} = P_{jt} [OF_{j0t} + \sum_{i=1, i \neq j}^J \Pi_{jit} F_{jit}] \quad (11)$$

3.5 Existence and Uniqueness of Equilibrium

We have J^2 offer probabilities and J market clearing conditions. Moreover, the system of equations which defines the equilibrium is linear in the offer probabilities. Hence we have the following result:

Proposition 1 *An equilibrium for the economy exists if the linear system of equations defined in (11) has a solution. These solutions (generically) exist since the number of equations is smaller than the number of unknowns (Strang, 1988).*

Proposition 1 implies that the housing authority has a lot of freedom to pursue its own objectives. There are many policies that are consistent with equilibrium in the public housing market. The market clearing conditions alone do not uniquely determine the offer probabilities.

To obtain a unique solution to this system of equations, we need to impose additional assumptions. It is plausible that the housing authority does not discriminate based on current residence and uses the same odds ratio for insiders and outsiders. We therefore assume that:

Assumption 5 *Offer probabilities do not depend on current residence:*

$$\Pi_{ijt} = \Pi_{jt} \quad (12)$$

The odds ratios are the same for household inside and outside of public housing:

$$\Pi_{0jt} = R_{0t} \Pi_{jt} \quad (13)$$

Note that $R_{0t} > 1$ measures the relative degree of preferential treatment that is given to outsiders, i.e. if $R_{0t} > 1$, then households from the outside get preferential treatment over households that are already in public housing. Substituting Assumption 5 into the definition of equilibrium, we obtain:

$$P_{0t} R_{0t} \Pi_{jt} F_{0jt} + \sum_{i \neq j} P_{it} \Pi_{jt} F_{ijt} = P_{jt} [OF_{j0t} + \sum_{i \neq j} \Pi_{it} F_{jit}] \quad (14)$$

which is a system of J equations in $J + 1$ unknowns. Thus the equilibrium conditions define the offer probabilities up to the factor R_{0t} . We thus have shown the following result:

Proposition 2 *For each value of R_{0t} , there exists a unique housing equilibrium equilibrium with rationing.*

In summary we have developed an equilibrium model of public housing that can generate rationing and excess demand in equilibrium. The model can also explain transfers within public housing since housing communities are treated as differentiated products. The model captures the fact that some households voluntarily leave public housing as their economic well-being improves. Exit rates from public housing also depend on the attractiveness of public housing relative to housing opportunities in the private market.

4 Identification and Estimation

We have discussed in Section 2 that our empirical approach draws on two samples, one of them is a choice based sample. One econometric problem is that both samples only allow the econometrician to observe a subset of the full choice set. Nevertheless, there is sufficient overlap in the choice sets such that the model can be estimated using an enriched sampling maximum likelihood estimator proposed by Cosslett (1981). To develop our empirical strategy, we need to a) introduce a parametrization of our model, b) derive the conditional choice probabilities; c) develop the ESML estimator; d) discuss the role that equilibrium conditions play in establishing identification of the model, e) show that our approach works

in a Monte Carlo study when the data generating process is known. We will discuss each of these points in this section.

4.1 A Parametrization

We assume that the utility associate with community j is given by

$$u_{jt} = \gamma_j + \beta \ln(y_{jt}) + \delta x_t + mc \mathbb{1}\{d_t \neq d_{t-1}\} + \epsilon_{jt} \quad j = 1, \dots, J \quad (15)$$

The utility associated with the outside option is normalized to be equal to:

$$u_{0t} = \ln(y_{0t}) + mc \mathbb{1}\{d_t \neq d_{t-1}\} + \epsilon_{0t} \quad (16)$$

where y_{jt} denotes household net income, mc moving costs, and the γ_j are community specific fixed effects. Note that households that live in public housing typically pay 30 % of their incomes in rent. As a consequence net income is choice specific due to the implicit tax. As income increases, living outside of public housing should become more attractive. We would, therefore, expect that $\beta < 1$. The community specific fixed effects capture the fact that there are observed and unobserved differences among the public housing communities. The specification also accounts for (psychic) moving costs. the idiosyncratic shocks account for factors not observed by the econometrician. We follow McFadden (1973) and assume that the ϵ 's are i.i.d. Type I extreme value distributed.

4.2 Conditional Choice Probabilities for the Choice Based Sample

Our main data set is a choice based sample since we only observe households that have chosen to live in one of the housing communities at time t . A household that lived in community j at the end of the last time period, has three options: 1) the household moves back to the private housing market; 2) the household moves to a different housing community; 3) the household stays in its current community j . The probability of moving to the private sector is then defined:

$$Pr\{d_{0t} = 1 | d_{jt-1} = 1, x_t\} = \sum_{k=1, k \neq j}^J \Pi_{jkt} \frac{\exp(u_0(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))}$$

$$+ \Pi_{jjt} \frac{\exp(u_0(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \quad (17)$$

The probability of moving from community j to community k is given by:

$$Pr\{d_{kt} = 1 | d_{jt-1} = 1, x_t\} = P_{jkt} \frac{\exp(u_k(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} \quad (18)$$

and the probability of staying in community j is given by:

$$Pr\{d_{jt} = 1 | d_{jt-1} = 1, x_t\} = \sum_{k=1, k \neq j}^J \Pi_{jkt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} + \Pi_{jjt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \quad (19)$$

Finally, we also observe new entrants into public housing. The probability of observing a new household in community j is

$$Pr\{d_{jt} = 1 | d_{0t-1} = 1, x_t\} = \Pi_{0jt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \quad (20)$$

The conditional choice probabilities for the choice based sample are thus defined by equations (17), (18), (19) and (20).

4.3 Conditional Choice Probabilities for the Random Sample

Our second sample is a random sample. The main econometric problem is that the SIPP does not allow us to identify the exact housing community in which a household lives. As a consequence we only observe a coarser version of the choice set in the SIPP. For households that are currently not living in public housing, we have we have two possible outcomes: 1) the household stays in private housing; 2) the household moves to a public housing unit.

The probability of moving to any of the J public housing communities is given by:

$$Pr\{d_{0t} = 0 | d_{0t-1} = 1, x_t\} = \sum_{j=1}^J \Pi_{0jt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \quad (21)$$

Note that (21) is obtained by summing the probabilities in (20) over all possible choices.

Similarly, the probability of staying in private housing is defined:

$$Pr\{d_{0t} = 1 | d_{0t-1} = 1, x_t\} = 1 - \sum_{j=1}^J \Pi_{0jt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))} \quad (22)$$

Note that we do not observe whether the household obtained an offer and we also do not observe to which housing unit it moved, if it decided to move.

Next consider a household that currently lives in public housing. Again there are two possible outcomes: 1) the household moves back to private housing; 2) the household stays in public housing. Consider the first case, in which the household moves back to private housing. Now we do not observe in the SIPP in which unit the household live. However, we can compute relative frequencies based on the HACP which assign probabilities to each community type. Let us denote these probabilities by $Pr\{d_{jt-1} = 1|d_{0t-1} = 0, x_t\}$. The choice probability conditional on living in community j is given by equation (17). Summing over all J housing units and properly weighting each conditional choice probability, implies that the probability of moving out of public housing is then:

$$Pr\{d_{0t} = 1|d_{0t-1} = 0, x_t\} = \sum_{j=1}^J Pr\{d_{0t} = 1|d_{jt-1} = 1, x_t\}Pr\{d_{jt-1} = 1|d_{0t-1} = 0, x_t\} \quad (23)$$

Next consider the case in which a household stays in public housing. In the SIPP we cannot distinguish between the case in which a household stays in the same community or moves to a different housing community within public housing. Thus conditional on living in community j , the probability of staying in public housing is the sum of the probabilities in equations (18) and (19), i.e. the probability of staying conditional on living in j at the end of the previous period is

$$Pr\{d_{0t} = 0|d_{jt-1} = 1, x_t\} = Pr\{d_{jt} = 1|d_{jt-1} = 1, x_t\} + \sum_{k=1, k \neq j}^J Pr\{d_{kt} = 1|d_{jt-1} = 1, x_t\} \quad (24)$$

Summing over all J housing units and properly weighting each conditional choice probability, implies that the probability of staying in public housing is then:

$$Pr\{d_{0t} = 0|d_{0t-1} = 0, x_t\} = \sum_{j=1}^J Pr\{d_{0t} = 0|d_{jt-1} = 1, x_t\}Pr\{d_{jt-1} = 1|d_{0t-1} = 0, x_t\} \quad (25)$$

The conditional choice probabilities for the SIPP are thus defined by equations (21), (22), (23) and (25).

4.4 The Likelihood Function under Enriched Sampling

To compute the likelihood function we need to take into account the fact that the SIPP is a random sample while the HACP is a choice based sample in which households only chose from a subset of the available options. This sampling scheme is also called enriched sampling as discussed in detail by Cosslett (1978, 1981).²³

Let us denote the corresponding sample sizes with N_1 and N_2 . Similarly, let T_1 and T_2 denote the length of the two panels. Observations are assumed to be independent across samples, i.e. that assumption basically rules out sampling of the same household in both data sets. The joint likelihood function of observing the SIPP and the HACP is thus the product of the two likelihood functions

$$L = L_1 L_2 \quad (26)$$

The likelihood associated with the SIPP sample L_1 is given by:

$$L_1 = \prod_{i=1}^{N_1} \prod_{t=1}^{T_1} l_{1nt} \quad (27)$$

where l_{1nt} is given by

$$l_{1nt} = [Pr\{d_{0nt} = 0 | d_{0nt-1}, x_{nt}\}]^{1-d_{0nt}} [Pr\{d_{0nt} = 1 | d_{0nt-1}, x_{nt}\}]^{d_{0nt}} f(x_{nt}, d_{nt-1}) \quad (28)$$

The likelihood for the HACP sample L_2 is defined:

$$L_2 = \prod_{i=1}^{N_2} \prod_{t=1}^{T_2} \frac{Pr\{d_{jnt} = 1 | d_{nt-1}, x_{nt}\} f(x_{nt}, d_{nt-1})}{\tilde{Q}_t(J)} \quad (29)$$

where

$$\tilde{Q}_t(J) = \sum_{j=1}^J Q_t(j) \quad (30)$$

²³Notice that our sampling scheme satisfies assumptions 9 and 10 in Cosslett (1981). As a consequence the model is identified if all offer probabilities are equal to one. We discuss the case of non-degenerate offer probabilities below.

$Q_t(j)$ is the unconditional probability that choice j is chosen that is defined as.

$$\begin{aligned} Q_t(j) &= \sum_{j=1}^J \int Pr\{d_{jnt} = 1 | d_{t-1}, x_t\} f(x_t, d_{t-1}) dx_t d_{t-1} \\ &= \sum_{j=1}^J \int \sum_{i=0}^J Pr\{d_{jnt} = 1 | d_{it-1} = 1, x_t\} f(x_t | d_{it-1} = 1) Pr\{d_{it-1} = 1\} dx_t \end{aligned} \quad (31)$$

We assume $f(x_t, d_{t-1}, \theta)$ is known up to finite vector of parameters θ and treat the the $Q_t(j)$ as unknown. We then define our enriched sampled maximum likelihood estimator (ESMLE) as the argument that maximizes equation (26).²⁴

4.5 Imposing the Equilibrium Constraints

One problem associated with the ESML estimator above is that the offer probabilities are not separately identified from the choice specific intercepts.²⁵ Having access to a random sample with the full choice set does not solve this problem. The key identification problem arises because housing quality is unobserved. This identification problem is similar to the identification problem encountered in job search models. However, there is one important difference. In labor search models, we observe the wage if a job is accepted by a worker. As a consequence, the standard “identification at infinity argument” can be used in occupational choice models to establish identification.²⁶ In our model, housing quality is ultimately unobserved. Hence, we need a different identification strategy. One way to resolve this identification problem is to use the restrictions incorporated in the equilibrium conditions in equations (14). Solving this linear system of equation, we can express the offer probabilities as functions of the observed initial distribution of households (P_i 's) and the

²⁴If the $Q_t(j)$'s are known, we can define a constrained enriched sampled maximum likelihood estimator (CESMLE) as the argument which maximizes equation (26) subject to the J constraints in equation (31). Finally, one could follow Cosslett (1978,1981) and treat $f(x_t, d_{t-1})$ as unknown and then define Pseudo MLE by concentrating out the weights that characterize the empirical likelihood of the data. These estimators extend the standard choice based estimators discussed in Manski and Lerman. (1977).

²⁵A simple example which illustrates these identification problems is available from the authors.

²⁶See, for example, Heckman and Honore (1990) for a discussion of these techniques applied to the Roy model.

choice probabilities (F_{ij} 's) which only depend on the structural parameters of the utility function. Imposing the equilibrium conditions thus resolves the key identification problem encountered in this model.

4.6 Evidence from A Monte Carlo Experiment

To provide some additional insights into our approach to identification and estimation, we have conducted a small Monte Carlo study.²⁷ The Monte Carlo study shows that our estimator works well under random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimate for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward in samples with 2000 observations. Larger samples help reduce the estimation bias. Imposing the equilibrium conditions works well and established identification. The estimates of the offer probabilities that are implied by the structural parameters of the model are accurate.

5 Empirical Results

We implemented our estimator for a number of different model specifications.²⁸ Table 5 reports the parameter estimates and estimated standard errors for the four models. In column I, we report the estimates of our baseline model. In column II, we estimate a richer model that controls for observed heterogeneity accounting for gender, race, age, and number of children as regressors. These regressors that are observed in the SIPP and the HACP. This model is our preferred specification. In column III we only include households in the

²⁷Details are reported in Appendix A.

²⁸In all models, we use the empirical demographic distributions to estimate $f(x_{nt}, d_{nt-1})$. Race (black, white) and age (senior, non-senior) are modeled as a multivariate distribution; sex is a binomial conditional on race-age; number of children is a multinomial conditional on sex and race-age; income is a truncated normal based on number of children, sex, and race-age. We fit a logit model to estimate $Pr\{d_{jt-1} = 1 | d_{0t-1} = 0, x_t\}$, which is needed in equations (23), (24), and (25) for the SIPP likelihood. We calibrate R_0 based on the observed ratios of mobility for households inside and outside of public housing.

SIPP that live in Pittsburgh. We find that there are small differences between the point estimates in columns II and III. Using the smaller Pittsburgh subsample largely affects the precision of the estimates. In column IV we add disability status as an additional regressor. Unfortunately the SIPP uses a much more restrictive definition of disability than the HACP and the U.S. Census. However, the results are similar.

Table 5: Parameter Estimates

	I		II		III		IV	
income	0.403	(0.028)	0.329	(0.028)	0.327	(0.0278)	0.300	(0.029)
moving cost	-3.506	(0.017)	-3.186	(0.017)	-3.203	(0.036)	-3.445	(0.263)
black, non-senior			1.222	(0.071)	1.221	(0.107)	1.181	(0.020)
white, senior			0.209	(0.113)	0.209	(0.152)	-0.080	(0.072)
black, senior			1.000	(0.101)	1.000	(0.138)	1.050	(0.116)
kids			-0.315	(0.123)	-0.317	(0.185)	0.414	(0.103)
female			0.053	(0.061)	0.053	(0.090)	-0.286	(0.151)
female, senior			-0.174	(0.094)	-0.173	(0.121)	0.116	(0.055)
female, kids			0.426	(0.130)	0.424	(0.195)	1.006	(0.091)
disabled							1.719	(0.157)
log likelihood	-690,071		-688,796		-687,796		-687,706	

Estimated standard errors are given in parenthesis.

We find that blacks have stronger preferences for public housing than whites. This result is largely driven by the fact that black households are overrepresented in public housing in Pittsburgh. We also find that age has an impact. Male seniors have a higher utility for public housing than female seniors. Females with children have a higher utility for public housing than other households, but fathers or married couples with children have lower utility for public housing than those without children. We also find that there are significant moving costs that constrain potential relocations of households. We illustrate

the importance of moving costs in our policy analysis discussed below.

The income coefficient shows that there are strong incentives for households to leave public housing as their income grows larger. This finding is consistent with the fact that there are only a few higher income household in our sample that live in public housing. Out of 4040 HACP households that we study, there are only 52 households that, at some time during the study, exceed the income eligibility limit of approximately \$45,000.²⁹ Most of these households are headed by a single, black female.

We also estimate community specific fixed effects which are not reported in the table above. Our findings suggest that there is significant heterogeneity among public housing communities. Smaller communities are in general more desirable than larger communities. We assign these fixed effects to the 34 communities in the full sample, we can correlated our estimates with observed amenities. We find that these fixed effects are positively correlated with median neighborhood income (.60), school quality (.17), home-ownership (.43), and percent white (.56). They are negatively correlated with crime (-.20) and vacant housing (-.29). We thus conclude that our community fixed effect estimates, at least, partially capture amenity values of the neighborhood.

Next we analyze the goodness of fit of our model. One measure of goodness of fit is to compare the residency distribution predicted by the model to the actual residency distribution observed in the sample. We find that that the predictions that are based on our preferred model are accurate. Our model, thus, matches the unconditional distributions of households among choices well. A more challenging exercise is to predict the composition of the housing communities using our model. We focus on the composition by gender and family status conditioning on race. The results are summarized in Table 6. The findings are by and large encouraging. Our model explains the demographic compositions of all communities well. It slightly over-predicts the average incomes of white households in most housing communities. However, our estimates are well within standard confidence intervals.

²⁹Note that this limit depends on year and size of household.

Table 6: Actual vs Estimated Composition of Communities

		Private	PH1	PH2	PH3	PH4	PH5	PH6
% Black	Observed	0.24	0.98	0.94	0.90	0.97	0.56	0.55
	Estimated	0.26	0.95	0.92	0.9	0.95	0.51	0.56
% Female	Observed	0.67 / 0.53	0.85 / 0.88	0.89 / 0.75	0.93 / 1.00	0.84 / 0.67	0.63 / 0.53	0.66 / 0.68
	Estimated	0.67 / 0.53	0.82 / 0.67	0.87 / 0.71	0.93 / 0.83	0.84 / 0.64	0.57 / 0.48	0.67 / 0.66
% Have Kids	Observed	0.46 / 0.24	0.55 / 0.64	0.62 / 0.43	0.62 / 0.38	0.58 / 0.1	0 / 0	0 / 0
	Estimated	0.42 / 0.24	0.49 / 0.28	0.57 / 0.36	0.60 / 0.37	0.59 / 0.19	0.06 / 0.02	0.05 / 0.02
Income	Observed	19.3 / 21.0	8.4 / 7.2	12.3 / 12.9	14.1 / 10.3	9.9 / 11.3	9.1 / 8.5	9.3 / 9.8
	Estimated	19.3 / 21.5	8.5 / 6.2	12.3 / 8.1	12.6 / 7.5	9.9 / 8.1	8.3 / 8.0	9.4 / 9.9

Composition Shown by Race black / white.

Finally, we compare the observed mobility with the mobility generated under the model. With the model parameters from our preferred model, the predicted number of move-ins during this whole sample is 1796. The actual number is 1581. The predicted move-outs 2273 (actual is 2106). Finally the predicted number of transfers is 374 compared to 349 observed in the data.³⁰

6 Quantifying the Excess Demand in Equilibrium

One main objective of the empirical analysis is to quantify the importance of rationing that is present in public housing markets. Based on the parameter estimates of our preferred model in column II we estimate the fraction of the population that would like to move into public housing if it were in the choice set. This fraction varies by quarter due to quarterly differences in income and demographic heterogeneity. Table 7 shows the percent willing to move for the 12th quarter (a quarter in the middle of the study).

Table 7: Percent of Households in Community i who would accept an offer to move to j

	Would move to:						
Current Residence:	Private	PH1	PH2	PH3	PH4	PH5	PH6
Private		0.006	0.012	0.009	0.008	0.009	0.012
PH1	0.080		0.067	0.054	0.044	0.055	0.071
PH2	0.063	0.020		0.029	0.023	0.029	0.039
PH3	0.075	0.023	0.043		0.028	0.035	0.045
PH4	0.077	0.031	0.056	0.045		0.046	0.059
PH5	0.102	0.022	0.041	0.032	0.026		0.043
PH6	0.085	0.019	0.034	0.027	0.022	0.028	

We find that there are serious supply restrictions which prevent the mobility of those

³⁰Some periods in the HACP data were eliminated. Only quarters overlapping with the SIPP data were included in the estimation.

willing to transfer or move into public housing. For example, combining the percent willing to move with the number of available units, 14.1 families would like to move into public housing for every family that wishes to move out. This ratio varies from 3.7 to 72.7 among the different housing communities. The fraction willing to move into a public housing unit largely depends on the community specific fixed effects and thus reflects the attractiveness of the housing community. However, it also depends the characteristics of eligible households. For example, we find that older households are more willing to move from the private sector to public housing communities.

7 Policy Implications

To study the policy implications of our model, we consider demolishing some of the least attractive public housing units. Jacob (2004) considers the impact of such demolitions in Chicago and finds few positive effects on a variety of outcomes. Here we instead focus on analyzing how demolitions affect the demand for public housing, the composition of housing communities and the degree of rationing. We implement this policy experiment by demolishing the first public housing unit (PH 1). Recall these are large units that are primarily used by households whose head is a single, black female. These units have the lowest fixed effect parameter.

We consider the demolition of public housing community 1 during the third period of a 12-quarter study. We use the estimates based on our preferred model in column II of Table 5. To initialize, the demographic characteristics in the first quarter are the same as those observed in the data. It is well-known that these types of discrete choice models do not yield closed form solutions for compensating variations. We, therefore, follow McFadden (1989, 1995) and adopt a simulation based approach. An additional complication in our model is that we not only need to simulate draws from distributions of the error terms, but also from the equilibrium offer probabilities. For families of varying demographic characteristics, we compute the median compensating variation for an evicted household earning \$12,000 per year. We find that the estimates range from \$11,656 for a white male with kids to \$116,010

for a black female with kids. White households require lower compensation to leave public housing than black households. Overall, the estimates suggest that there may be significant welfare losses associated with demolishing existing units.³¹ The policy experiment shows a decline in overall welfare for low-income blacks. However for some low-income households earning more than \$12,000 a year, there is a small welfare gain. This finding suggests that a portion of families in public housing might be "stuck" in public housing because they are unable to pay the one-period moving costs; but after the moving costs are paid, they actually experience higher utility outside of public housing. Indeed, if we alter the model so that moving costs are zero but keep the other parameters the same, the probability that a black female-headed household earning \$12,000 voluntarily chooses to leave public housing increases from 9.5% to 71%.

We also study the impact of evicting all public housing households in PH 1 on the equilibrium offer probabilities. Compared to the baseline equilibrium, offer probabilities immediately decrease after the eviction because many evicted tenants wish to move back into public housing. Offer probabilities decrease 2.6% for PH 2, 12% PH 3 (medium and small family communities), 6.3% for PH 4 (mixed family and senior communities), and 16% for PH 5 and PH 6 (mostly senior communities). Over time, the composition of the remaining public housing communities changes. The public housing communities see an increase of 3% in black households and a 12% decrease in non-black households; there is a 1.3% increase in female-headed households and 2.2% increase in households with children. Average income in the public housing communities decreases 2% (and average income in private housing declines 1.3%).

³¹Of course, a full cost-benefit analysis would require the inclusion of the cost of maintaining these housing units as well as potential impacts of living in public housing on educational achievements and criminal outcomes.

8 Conclusions

We developed an equilibrium model that captures housing choices of low-income households. The model is based on the notion that private housing is in ample supply, while publicly provided subsidized housing is desirable and scarce. As a consequence demand of eligible households potentially exceeds available supply. At each point of time a fraction of households that used to live in public housing voluntarily leave public housing communities to rent in the private market. The housing authority can offer new eligible households access to the vacated units. In equilibrium the housing authority adjusts the offer probabilities so that the inflow into public housing equals the voluntary outflow.

We have also developed a maximum likelihood estimator for the parameters of the model. The estimator accounts for enriched sampling and imposes the equilibrium conditions in estimation. We have estimated the parameters of the model using data from the Pittsburgh metropolitan area. We find that our model fits the data well. In equilibrium, there is significant amount of rationing and excess demand for public housing. We have shown that the model can be used to evaluate some potentially important policy options.

While we view the research conducted in this paper as promising, there is ample scope for future work. One potentially interesting extension of our model would account for forward-looking behavior of households. In our model, households are myopic and maximize short term period utility. Extending the decision problem of the households to a dynamic framework is fairly straight forward.³² However, it is significantly more difficult to establish and characterize equilibrium in such a model. The main difficulty arises because offer probabilities determined by the housing authority enter the dynamic decision of the households in a nonlinear way. Forward looking households have to be concerned about the likelihood of receiving offers in the future. The value function associated with the different discrete options will, therefore, depend on these offer probabilities. As a consequence, the equilibrium offer probabilities can no longer be characterized by a solution to linear system of equations.

³²See, for example, Keane and Wolpin (1997) or Eckstein and Wolpin (1999).

Introducing dynamics in household locational choice will allow us to get a better understanding of mobility patterns, based on more realistic assumptions capturing the costs and benefits of mobility.³³ We view these issues as an important topic for future research.

³³Some early dynamic work include Benabou (1996a, 1996b), Durlauf (1996), Fernandez and Rogerson (1998), and Glomm and Lagunoff (1999). More recently Ortalo-Magne and Rady (2006) have worked on the role of uncertainty in housing prices on residential sorting. Epple, Romano, and Sieg (2009b) study the impact of limited mobility on residential within an overlapping generations model. Bayer, McMillan, Murphy, and Timmins (2007b) estimate a dynamic housing demand model. Murphy (2007) considers the dynamics of land conversion and housing development.

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A A Monte Carlo Study

Since our estimation procedure is non standard, we conducted a number of Monte Carlo studies to study the properties of the estimators when the true data generating process is known. Below we report the results for one specification that we tested.³⁴

Table 8: 95% Confidence Intervals of Estimation Error

Name	Variable	random sample	enriched sample
Fixed Effect PH1	γ_1	[-0.887, 1.763]	[-0.947, 1.763]
Fixed Effect PH2	γ_2	[-.8142, 1.585]	[-1.010, 1.585]
Fixed Effect PH3	γ_3	[-0.806, 1.744]	[-0.850, 1.744]
Beta	β	[-0.191, 0.079]	[-0.191, 0.082]
Offer Prob PH1	π_1	[-0.021 ,0.019]	[-0.020, 0.019]
Offer Prob PH2	π_2	[-0.043 ,0.050]	[-0.046, 0.055]
Offer Prob PH3	π_3	[-0.013 ,0.010]	[-0.013, 0.010]

In our Monte Carlo there is only one observed household characteristic ('income'). We assume that $f(x_t, d_{t-1})$ is log-normally distributed with known mean and variance. We consider a model with three public housing communities with $\gamma_1 = 7.6$, $\gamma_2 = 7.0$ and $\gamma_3 = 0.4$. We set the coefficient of income $\beta = 0.4$. We assign 30 % of the population to private housing, 24, 28, and 18 percent to the three housing communities. This implies that in equilibrium the offer probabilities are $\pi_1 = .11$, $\pi_2 = .24$ and $\pi_3 = 0.05$.

We consider the properties of the estimator above under two sampling designs: random sampling and enriched sampling. For each parameter vector, one hundred model simulations and estimations are completed, each with sample size 2000. Starting values are initially chosen from a uniform distribution between (0, 1) for β and between [0, 12] for the fixed effects, but any starting values that would lead to unreasonable offer probabilities (probabilities

³⁴More results for different parametrizations, sample sizes and sampling schemes are available upon request from the authors.

greater than 40%) are rejected. The table below summarizes the performance of the model and reports 95% confidence for the absolute error of parameter estimate and the implied offer probabilities.

In general we find that our estimator works well both under random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimate for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward in samples with 2000 observations. In general, larger samples help reduce the estimation bias. Imposing the equilibrium conditions seems to work well, and the estimates of the offer probabilities that are implied by the structural parameters of the model are accurate.

B The Extended SIPP Sample

In addition to the Pittsburgh sample, we also construct a larger sample adding data from 13 metropolitan areas that have similar ratios of public housing units per household as Pittsburgh. Table 9 provides some summary statistics of these MSA's.

Table 9: Urban Areas Included in Sample

City	Eligible for Public Housing	Median Income	Unemployment Rate	Minority	Fair Market Rent 2001
Pittsburgh	.0546	37467	4.4%	10%	476
Columbus	.0384	44782	2.7%	19%	471
Allentown	.0375	43098	4.2%	10%	511
Albany	.0373	43250	3.4%	10%	494
Dayton	.0372	41550	4.5%	18%	389
Buffalo	.0339	38488	5.3%	16%	453
Scranton	.0607	34161	5.6%	3%	408
St. Louis	.0169	44437	3.5%	22%	429
Madison	.0124	49223	1.7%	11%	559
Detroit	.0159	49160	3.9%	27%	598
Cleveland	.0291	42215	4.2%	21%	555
Cincinnati	.0109	44914	3.5%	15%	416
Philadelphia	.0266	47528	4.1%	27%	657
Milwaukee	.0193	46132	3.1%	22%	504

Table 9 reports the MSA's ratio of public housing units to households eligible for public housing. We also show the 1999 MSA median income, 1999 unemployment rate, and the HUD-determined 2001 fair market rent for a one-bedroom unit.³⁵ Table 9 shows that

³⁵The number of public housing units is taken from the HUD 1998 Picture of Subsidized Housing. Percent minority and median incomes are from the 2000 Census. Unemployment is from The Real Estate Center at Texas A& M University. Fair Market Rents are published on the HUD website.

Pittsburgh is representative of many other large urban areas in the Northeast and Midwest that face similar challenges in providing affordable housing for low-income households.